

Mechanism of Magnetic Flux Loss in Molecular CloudsTakenori Nakano and Ryoichi Nishi¹

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ABSTRACT

We investigate the detailed processes working in the drift of magnetic fields in molecular clouds. To the frictional force, whereby the magnetic force is transmitted to neutral molecules, ions contribute more than half only at cloud densities $n_{\text{H}} \lesssim 10^4 \text{ cm}^{-3}$, and charged grains contribute more than about 90% at $n_{\text{H}} \gtrsim 10^6 \text{ cm}^{-3}$. Thus grains play a decisive role in the process of magnetic flux loss. Approximating the flux loss time t_B by a power law $t_B \propto B^{-\gamma}$, where B is the mean field strength in the cloud, we find $\gamma \approx 2$, characteristic to ambipolar diffusion, only at $n_{\text{H}} \lesssim 10^7 \text{ cm}^{-3}$ where ions and smallest grains are pretty well frozen to magnetic fields. At $n_{\text{H}} > 10^7 \text{ cm}^{-3}$, γ decreases steeply with n_{H} , and finally at $n_{\text{H}} \approx n_{\text{dec}} \approx \text{a few} \times 10^{11} \text{ cm}^{-3}$, where magnetic fields effectively decouple from the gas, $\gamma \ll 1$ is attained, reminiscent of Ohmic dissipation, though flux loss occurs about 10 times faster than by pure Ohmic dissipation. Because even ions are not very well frozen at $n_{\text{H}} > 10^7 \text{ cm}^{-3}$, ions and grains drift slower than magnetic fields. This insufficient freezing makes t_B more and more insensitive to B as n_{H} increases. Ohmic dissipation is dominant only at $n_{\text{H}} \gtrsim 1 \times 10^{12} \text{ cm}^{-3}$. While ions and electrons drift in the direction of magnetic force at all densities, grains of opposite charges drift in opposite directions at high densities, where grains are major contributors to the frictional force. Although magnetic flux loss occurs significantly faster than by Ohmic dissipation even at very high densities as $n_{\text{H}} \approx n_{\text{dec}}$, the process going on at high densities is quite different from ambipolar diffusion in which particles of opposite charges are supposed to drift as one unit.

Subject headings: ISM: clouds — ISM: dust — ISM: magnetic fields — magnetic fields — plasmas — stars: formation

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1. Introduction

Magnetic fields of interstellar molecular clouds are widely believed to have significant effect on star formation. For an oblate cloud or cloud core of mass M contracted along field lines to some extent, there is a critical value for its magnetic flux Φ given by

$$\Phi_{\text{cr}} = f_{\phi} G^{1/2} M, \quad (1)$$

where G is the gravitational constant and f_{ϕ} is a dimensionless constant. A cloud (core) with $\Phi < \Phi_{\text{cr}}$ cannot be kept in hydrostatic equilibrium by the magnetic force alone (this state is widely called magnetically supercritical), and one with $\Phi > \Phi_{\text{cr}}$ (magnetically subcritical) can be in equilibrium if its expansion is suppressed by external magnetic fields. Applying the virial theorem to such clouds, Strittmatter (1966) found f_{ϕ} between 4.9 and 9.4 depending on the flatness of the cloud. From some numerical cloud models Mouschovias & Spitzer (1976) obtained $f_{\phi} \approx 8.0$, and Tomisaka, Ikeuchi, & Nakamura (1988) got $f_{\phi} \approx 8.3$. Li & Shu (1996) found $f_{\phi} = 2\pi$ for self-similar, singular, isothermal clouds.

We can define the critical magnetic field strength of the cloud (core) by

$$B_{\text{cr}} = \frac{\Phi_{\text{cr}}}{\pi R^2} = f_{\phi} G^{1/2} \Sigma, \quad (2)$$

where R is the radius of the cloud (core) perpendicular to the mean magnetic field direction and $\Sigma = M/\pi R^2$ is the mean column density of the cloud (core) along field lines. Of course $B \geq B_{\text{cr}}$ is equivalent to $\Phi \geq \Phi_{\text{cr}}$, where $B = \Phi/\pi R^2$ is the mean field strength in the cloud (core). Nakano & Nakamura (1978) found that isothermal disks penetrated by uniform magnetic fields \mathbf{B} perpendicular to the disk layers are gravitationally unstable only when $B < B_{\text{cr}}$, where B_{cr} is given by equation (2) with $f_{\phi} = 2\pi$. The critical wavelength, below which the disk is unstable, decreases as B/B_{cr} decreases (Nakano 1988). These mean that with perturbations at least the disk as a whole can contract perpendicular to field lines when $B < B_{\text{cr}}$.

While magnetic fluxes of clouds estimated by observations are not much smaller than their critical values, or $\Phi/\Phi_{\text{cr}} \sim 1$ (e.g., Crutcher 1999), magnetic stars with mean surface fields of 1 kG to 30 kG have ratios $\Phi/\Phi_{\text{cr}} \approx 10^{-5} - 10^{-3}$ (Nakano 1983). For ordinary stars like the Sun with mean surface fields of ~ 1 G, Φ/Φ_{cr} is as small as 10^{-8} . This suggests that cloud cores must lose most of their initial magnetic fluxes at some stages of star formation. At what stages and by what mechanisms? This is called the magnetic flux problem in star formation (e.g., Nakano 1984).

Ohmic dissipation is too slow to dissipate magnetic fields in molecular clouds of ordinary densities. For example, for the ionization fraction 10^{-8} at density 10^5 cm^{-3} (Figure 1) and the length scale of magnetic fields, 0.1 pc, about the Jeans length at this density and temperature 10 K, we obtain the Ohmic dissipation time 10^{15} yr, larger than the age of the universe by orders of magnitude. Mestel & Spitzer (1956) found another process of decreasing magnetic flux, which is now widely called ambipolar diffusion (sometimes called plasma drift; see Appendix C for the

terminology) and is much more efficient than Ohmic dissipation in molecular clouds of ordinary densities. In this process ions (and electrons), which are well frozen to magnetic fields, drift in the sea of neutral molecules together with magnetic fields at a terminal velocity with which the magnetic force balances with the frictional force exerted by the neutrals.

Because some dust grains in clouds are electrically charged and interact with magnetic fields, they contribute to controlling the drift of magnetic fields as well as ions. However, because grains are not so strongly coupled with magnetic fields as ions due to their large masses, complete freezing is not a good approximation in most situations. More accurate treatment is required for their motion (Elmegreen 1979). Moreover, the size and the mass of grains are distributed in wide ranges (e.g., Mathis, Rumpl, & Nordsieck 1977, referred to as MRN hereafter), and the degree of freezing depends sensitively on their masses. More accurate treatment is also necessary for ions because even ions are not well frozen at high densities. Nakano (1984) and Nakano & Umebayashi (1986, referred to as NU86 hereafter) formulated a method of describing the drift of magnetic fields in clouds containing any kinds of charged particles which are coupled with magnetic fields at arbitrary strengths.

Using this formalism we investigated the time scale of magnetic flux loss, t_B , from a major part of a cloud (core) as a function of the mean density of the cloud (core) (Nakano 1984; NU86; Umebayashi & Nakano 1990; Nishi, Nakano, & Umebayashi 1991, referred to as NNU91 hereafter). We have found that there is a critical value $n_{\text{dec}} \approx 10^{11}$ hydrogen nuclei per cm^3 for the density of the cloud, n_H , at which t_B is equal to the free-fall time t_{ff} of the cloud (core); $t_B < t_{\text{ff}}$ holds only at $n_H > n_{\text{dec}}$, and $t_B \gg t_{\text{ff}}$ at $n_H \ll n_{\text{dec}}$. We have called this the decoupling density because extensive flux loss occurs only at $n_H \gtrsim n_{\text{dec}}$.

Our previous results show that at $10^3 \text{ cm}^{-3} \lesssim n_H \lesssim 10^7 \text{ cm}^{-3}$, t_B is 10 to 10^2 times t_{ff} for $B = B_{\text{cr}}$ and a relation $t_B \propto B^{-2}$ approximately holds at least for $B_{\text{cr}} \geq B \gtrsim 0.1 B_{\text{cr}}$. The relation $t_B \propto B^{-2}$ is characteristic to ambipolar diffusion which occurs when the dominant charged particles are well frozen to magnetic fields (see § 2.2 for details). As the density increases at $n_H \gtrsim 10^7 \text{ cm}^{-3}$, t_B becomes more and more insensitive to B , and finally at $n_H \approx n_{\text{dec}}$, t_B becomes almost independent of B , reminiscent of Ohmic dissipation. Recently Desch & Mouschovias (2001) wrote that ambipolar diffusion was the dominant process even at densities several orders of magnitude higher than n_{dec} . These results may cause confusion. Besides, the dependence of t_B on n_H and B described above is rather complicated. It would be necessary to clarify what is going on especially at high densities. Furthermore, although it was pointed out that grains play an important role in the process of magnetic flux loss, it does not seem to be widely recognized how important they are.

The purpose of this paper is to clarify detailed mechanisms operating in the loss of magnetic flux in molecular clouds especially at high densities and to show how the grains behave. In § 2 we summarize some of the formulae obtained by Nakano (1984) and NU86, which will be used in this paper. In § 3 we show numerical results and analyze in detail the processes going on especially at

high densities. Discussion is made in §4, and summary is given in §5. In Appendix A we show another method of the formulation than that of Nakano (1984) and NU86.

2. Drift of Charged Particles and Magnetic Fields

2.1. Formulae

We summarize some of the formulae obtained by Nakano (1984) and NU86, which will be used in this paper. We consider a cloud which is composed mainly of neutral molecules and atoms but contains a slight amount of charged particles such as various atomic and molecular ions, electrons, and grains of various sizes and charges.

Because each kind of charged particles are scarce, we can neglect in its equation of motion the pressure force, the gravity, and the inertia term compared with the Lorentz force and the frictional force exerted by the neutrals. With this approximation we obtain the drift velocity (velocity of the guiding center) \mathbf{v}_λ of an arbitrary particle λ of mass m_λ and electric charge q_λ relative to the neutrals. We adopt the local Cartesian coordinate system with the z -axis parallel to the local magnetic field vector \mathbf{B} and the x -axis along the magnetic force $\mathbf{j} \times \mathbf{B}/c$, where $\mathbf{j} = (c/4\pi)\nabla \times \mathbf{B}$ is the electric current density and c is the light velocity. The components of the drift velocity \mathbf{v}_λ are given by

$$v_{\lambda x} = \frac{(\tau_\lambda \omega_\lambda)^2}{1 + (\tau_\lambda \omega_\lambda)^2} \frac{1}{A_1^2 + A_2^2} \left(A_1 + \frac{A_2}{\tau_\lambda \omega_\lambda} \right) \frac{1}{c} |\mathbf{j} \times \mathbf{B}|, \quad (3)$$

$$v_{\lambda y} = \frac{(\tau_\lambda \omega_\lambda)^2}{1 + (\tau_\lambda \omega_\lambda)^2} \frac{1}{A_1^2 + A_2^2} \left(\frac{A_1}{\tau_\lambda \omega_\lambda} - A_2 \right) \frac{1}{c} |\mathbf{j} \times \mathbf{B}|. \quad (4)$$

Here, τ_λ is the viscous damping time of the motion of particle λ in the sea of the neutrals, whose expressions are given, e.g., by Nakano (1984) for various particles,

$$\omega_\lambda = \frac{q_\lambda B}{m_\lambda c} \quad (5)$$

is the gyrofrequency (defined to be negative for negatively charged particles),

$$A_1 = \sum_\nu \frac{\rho_\nu \tau_\nu \omega_\nu^2}{1 + (\tau_\nu \omega_\nu)^2} = \frac{B}{c} \sum_\nu \frac{n_\nu q_\nu \tau_\nu \omega_\nu}{1 + (\tau_\nu \omega_\nu)^2} = \left(\frac{B}{c} \right)^2 \sigma_P, \quad (6)$$

$$A_2 = \sum_\nu \frac{\rho_\nu \omega_\nu}{1 + (\tau_\nu \omega_\nu)^2} = \frac{B}{c} \sum_\nu \frac{n_\nu q_\nu}{1 + (\tau_\nu \omega_\nu)^2} = \left(\frac{B}{c} \right)^2 \sigma_H, \quad (7)$$

where ρ_ν and $n_\nu = \rho_\nu/m_\nu$ are the mass and number densities, respectively, of particle ν , and σ_P and σ_H are Pedersen and Hall conductivities, respectively (see Appendix A). Summation in equations (6) and (7) is for all kinds of charged particles; particles having different values of τ_ν or

ω_ν are different kinds. The component $v_{\lambda z}$ is not necessary because the drift along field lines has no effect on the magnetic flux loss.

The drift velocity of magnetic fields, \mathbf{v}_B , is defined as the velocity relative to the neutrals of each point on an arbitrary closed contour in the cloud with which the magnetic flux through the contour is conserved. This requires

$$\mathbf{E}_\perp + \frac{1}{c} \mathbf{v}_B \times \mathbf{B} = 0, \quad (8)$$

where \mathbf{E}_\perp is the component of the electric field \mathbf{E} perpendicular to \mathbf{B} in the frame moving with the neutrals. Using the relation between \mathbf{v}_λ and \mathbf{E} , or the equation of motion, we obtain

$$v_{Bx} = \frac{A_1}{A_1^2 + A_2^2} \frac{1}{c} |\mathbf{j} \times \mathbf{B}|, \quad (9)$$

$$v_{By} = -\frac{A_2}{A_1^2 + A_2^2} \frac{1}{c} |\mathbf{j} \times \mathbf{B}|. \quad (10)$$

The rate of magnetic flux loss is determined by v_{Bx} alone (NU86). This is self-evident for axisymmetric clouds because the local y -axis is in the azimuthal direction. Therefore, the subscript x of v_{Bx} and $v_{\lambda x}$ may be omitted in the following.

Using equations (3) and (4) we can calculate the electric current density in the (x, y) plane, $\mathbf{J} = \sum_\lambda n_\lambda q_\lambda \mathbf{v}_\lambda$. With the electrical neutrality relation $\sum_\lambda \rho_\lambda \omega_\lambda = 0$, we can easily show that $J_x = 0$, which is consistent with the definition of the x -axis, and that J_y is equal to the component perpendicular to \mathbf{B} of \mathbf{j} which appears in equations (3) and (4). Thus the formulation has been consistently done. Consistency of equations (9) and (10) with equations (3) and (4) can be confirmed by considering the motion of a test particle which is completely frozen to magnetic fields and drifts with magnetic fields. Taking a limit $|\tau_\lambda \omega_\lambda| \rightarrow \infty$ reduces equations (3) and (4) to equations (9) and (10), respectively, because A_1 and A_2 are not affected by the test particle.

In this formalism we have neglected the effect of charge fluctuation of grains caused by sticking of ions and electrons on their motion because Nakano & Umebayashi (1980) and NU86 had a rough estimation that this effect is small. More elaborate discussion on this effect will be given in § 4.2. We have also neglected the collision between charged particles, whose effect is negligibly small compared with the effect of their collision with the neutrals as shown in Appendix B.

With the approximation made on the motion of charged particles, the magnetic force on charged particles must balance with the frictional force exerted on them by the neutrals, or by components

$$\sum_\lambda \frac{\rho_\lambda v_{\lambda x}}{\tau_\lambda} = \frac{1}{c} |\mathbf{j} \times \mathbf{B}|, \quad (11)$$

$$\sum_\lambda \frac{\rho_\lambda v_{\lambda y}}{\tau_\lambda} = 0. \quad (12)$$

It is easy to confirm that \mathbf{v}_λ given by equations (3) and (4) satisfies these equations.

One may simply think that the quantity $|\tau_\lambda \omega_\lambda|$ would characterize the degree of freezing of particle λ to magnetic fields. However, we have from equations (3) and (9)

$$\frac{v_{\lambda x}}{v_{Bx}} = \frac{(\tau_\lambda \omega_\lambda)^2}{1 + (\tau_\lambda \omega_\lambda)^2} \left(1 + \frac{A_2}{A_1 \tau_\lambda \omega_\lambda} \right). \quad (13)$$

When $|\tau_\lambda \omega_\lambda| \gg 1$, the degree of freezing, or the relative drift velocity, is certainly determined by $\tau_\lambda \omega_\lambda$ alone as $v_{\lambda x}/v_{Bx} \approx (\tau_\lambda \omega_\lambda)^2/[1 + (\tau_\lambda \omega_\lambda)^2]$ because $|A_2|/A_1$ is at most several at least in the ranges of density and field strength covered by this paper. However, when $|\tau_\lambda \omega_\lambda|$ is not much larger than 1, the relative drift velocity may be greatly affected by the second term in the parentheses of equation (13), or by other charged particles. We shall show some examples of this effect in §§ 3.4 and 3.5.

Making use of equations (8), (9), and (10), NU86 showed that the electric field $\mathbf{E}_0 = \mathbf{E} - \mathbf{u}_n \times \mathbf{B}/c$ in the frame wherein the neutrals move with a velocity \mathbf{u}_n is given by

$$\mathbf{E}_0 = -\frac{1}{c} \mathbf{u}_n \times \mathbf{B} + \frac{1}{\sigma_{\parallel}} \mathbf{j} + \beta \mathbf{j} \times \mathbf{B} - \xi (\mathbf{j} \times \mathbf{B}) \times \mathbf{B}, \quad (14)$$

where

$$\sigma_{\parallel} = \sum_{\nu} \sigma_{\nu}, \quad \sigma_{\nu} = \frac{n_{\nu} q_{\nu}^2 \tau_{\nu}}{m_{\nu}} = \frac{c}{B} n_{\nu} q_{\nu} \tau_{\nu} \omega_{\nu} \quad (15)$$

is the electric conductivity parallel to magnetic field lines, and

$$\beta = \frac{B}{c^2} \frac{A_2}{A_1^2 + A_2^2}, \quad (16)$$

$$\xi = \frac{1}{c^2} \frac{A_1}{A_1^2 + A_2^2} - \frac{1}{B^2 \sigma_{\parallel}}. \quad (17)$$

The work $\mathbf{j} \cdot \mathbf{E}_0$ gives the dissipation rate of magnetic energy per unit volume. The first term on the right-hand side of equation (14) causes amplification of magnetic fields by fluid motion. The second term leads to Ohmic dissipation. The third term yields no work. The last term gives rise to the dissipation in excess of Ohmic dissipation; ambipolar diffusion in excess of Ohmic dissipation is given by this term when the dominant charged particles are well frozen to magnetic fields. Thus the ratio of the last term to the second term in equation (14)

$$D \equiv B^2 \xi \sigma_{\parallel} = \frac{\sigma_{\parallel} \sigma_P}{\sigma_P^2 + \sigma_H^2} - 1 \quad (18)$$

gives the excess dissipation relative to Ohmic dissipation (NU86).

In Appendix A we show another method of obtaining these formulae than that adopted by Nakano (1984) and NU86.

2.2. Limiting Cases

Because the drift velocity v_{Bx} is expressed in terms of the complicated quantities A_1 and A_2 , it would be worthwhile to show how magnetic fields drift in some limiting cases.

When the dominant charged particles are well frozen to magnetic fields, or $|\tau_\nu \omega_\nu| \gg 1$, equations (6) and (7) give $A_1 \approx \sum_\nu \rho_\nu / \tau_\nu \gg |A_2| \approx |\sum_\nu \rho_\nu / (\tau_\nu^2 \omega_\nu)|$, and thus these particles drift at almost the same velocity irrespective of their charges as seen from equation (3). These relations also reduce equation (9) to

$$v_{Bx} \sum_\nu \frac{\rho_\nu}{\tau_\nu} \approx \frac{1}{c} |\mathbf{j} \times \mathbf{B}|. \quad (19)$$

This can also be obtained by replacing $v_{\lambda x}$ with v_{Bx} in equation (11). Equation (19) means that the charged particles drift together with magnetic fields with the terminal velocity at which the magnetic force balances with the frictional force exerted by the neutrals. This is the generalization of ambipolar diffusion investigated first by Mestel & Spitzer (1956), who considered only ions as the transmitter of the magnetic force to the neutrals. Because the magnetic force balances with the gravitational force perpendicular to field lines when $B = B_{\text{cr}}$, the mean magnetic force in an oblate cloud with $B \leq B_{\text{cr}}$ can be given by

$$\frac{1}{c} |\mathbf{j} \times \mathbf{B}| \approx \left(\frac{B}{B_{\text{cr}}} \right)^2 \frac{\pi}{2} G \Sigma \rho, \quad (20)$$

where ρ and Σ are the mean density and the mean column density along field lines, respectively, of the cloud. From equations (19) and (20) we have $v_{Bx} \propto B^2$, and then the magnetic flux loss time from the major part of the cloud, defined as the time required to drift a length scale L of magnetic fields, $t_B \approx L/v_{Bx} \propto B^{-2}$. This is characteristic to ambipolar diffusion in magnetically supercritical clouds.

When ions are the major contributors to the left-hand side of equation (19), whose mass m_i is much larger than that of an H_2 molecule and whose viscous damping time is given by $\tau_i \approx m_i / (\rho \langle \sigma v \rangle_i)$ with the collision rate coefficient $\langle \sigma v \rangle_i \approx 1.5 \times 10^{-9} \text{ cm}^3 \text{ s}^{-1}$ (Nakano 1984), we have

$$t_B \approx \frac{n_i \langle \sigma v \rangle_i}{\pi G \rho} \left(\frac{B_{\text{cr}}}{B} \right)^2 \quad (21)$$

for nearly spherical clouds with $\rho \approx \Sigma / (2L)$ insofar as $B \leq B_{\text{cr}}$. This is a well-known expression of the ambipolar diffusion time for magnetically supercritical clouds, which is proportional to the ionization fraction n_i/n_{H} .

If there exists a magnetically subcritical cloud core ($B > B_{\text{cr}}$) in equilibrium owing to external magnetic fields which suppress its expansion, the magnetic force therein almost balances with the gravity, or $|\mathbf{j} \times \mathbf{B}|/c \approx \pi G \Sigma \rho / 2$ though this must be significantly smaller than $B^2 / (4\pi R)$, where R is the core radius. Substituting this relation into equation (19), we find that v_{Bx} and t_B are independent of B and equal to those at $B = B_{\text{cr}}$ as shown by Nakano (1998).

Another limiting case is where the dominant charged particles are not at all frozen to magnetic fields, or $|\tau_\nu \omega_\nu| \ll 1$. In this case we have

$$A_1 \approx \sum_\nu \rho_\nu \tau_\nu \omega_\nu^2 = \left(\frac{B}{c}\right)^2 \sigma_\parallel, \quad (22)$$

$$A_2 \approx \sum_\nu \rho_\nu \omega_\nu (1 - \tau_\nu^2 \omega_\nu^2) = - \sum_\nu \rho_\nu \tau_\nu^2 \omega_\nu^3. \quad (23)$$

Electrical neutrality derives the last expression of equation (23). Again we find $A_1 \gg |A_2|$. Taking $|\mathbf{j} \times \mathbf{B}|/c \approx B^2/(4\pi L)$, we obtain the drift time of magnetic fields

$$t_B \approx \frac{L}{v_{Bx}} \approx \frac{4\pi\sigma_\parallel}{c^2} L^2. \quad (24)$$

This is the well-known Ohmic dissipation time, which does not depend on the field strength.

Thus our formalism contains ambipolar diffusion and Ohmic dissipation as limiting cases.

3. Numerical Results

3.1. Cloud Model

We adopt the cloud model almost the same as NNU91. Because magnetized clouds can contract along field lines rather easily, the force balance may approximately hold between gravity and gas pressure along field lines even when they are contracting dynamically perpendicular to field lines (e.g., Scott & Black 1980; Nakamura, Hanawa, & Nakano 1995). In such clouds the mean density ρ and the mean column density Σ along field lines have a relation

$$\Sigma \approx \left(\frac{4kT\rho}{\pi G \mu m_H}\right)^{1/2} \approx 0.040 \left(\frac{n_H}{10^5 \text{ cm}^{-3}} \frac{T}{10 \text{ K}} \frac{2.37}{\mu}\right)^{1/2} \text{ g cm}^{-2}, \quad (25)$$

where T is the mean temperature of the cloud, k is the Boltzmann constant, μ is the mean molecular weight of the gas, and m_H is the mass of a hydrogen atom. Equation (25) gives the column density of an isothermal disk in equilibrium if ρ is half the density at the midplane. If magnetic fields are weak and the isothermal (spherical) cloud is on the verge of collapse (Ebert 1955; Bonnor 1956), the mean column density is 0.9 times that of equation (25). The half-thickness of the cloud is given by $Z \approx \Sigma/(2\rho)$, or

$$Z \approx \left(\frac{kT}{\pi G \mu m_H \rho}\right)^{1/2} \approx 0.027 \left(\frac{10^5 \text{ cm}^{-3}}{n_H} \frac{T}{10 \text{ K}} \frac{2.37}{\mu}\right)^{1/2} \text{ pc}. \quad (26)$$

Both Σ and Z are independent of the cloud mass.

3.2. Densities of Charged Particles

As the reaction scheme of determining densities of various charged particles, we adopt the same model as Umebayashi & Nakano (1990) and NNU91 though we have revised some of the rate coefficients according to Le Teuff, Millar, & Markwick (2000); the greatest change is for dissociative recombination of H_3^+ . In dense clouds sufficiently opaque to interstellar ultraviolet radiation, ions and free electrons are formed mainly by ionization of H_2 molecules and He atoms by cosmic rays. After some reactions in the gas phase and at grain surface they finally recombine each other. We determine the densities of various charged particles assuming steady state for all the reactions.

As cosmic rays go deep into a cloud, their intensity decreases by interaction with matter, and the ionization rate decreases exponentially as $\zeta = \zeta_0 \exp(-\chi/\chi_0)$, where ζ_0 is the ionization rate of an H_2 molecule at the cloud surface, χ is the depth in column density from the cloud surface, and $\chi_0 \approx 96 \text{ g cm}^{-2}$ is the attenuation length of the ionization rate (Umebayashi & Nakano 1981). We take $\chi = \Sigma/4$ because we are interested in the mean densities of various particles in the cloud. We also take into account the ionization by radioactive elements contained in the cloud at a rate $6.9 \times 10^{-23} \text{ s}^{-1}$ (Umebayashi & Nakano 1981), which is important only at $n_{\text{H}} \gtrsim 10^{15} \text{ cm}^{-3}$.

As charged particles we consider electrons e^- , atomic ions H^+ , He^+ , and C^+ , metallic ions such as Mg^+ , Si^+ , and Fe^+ , which we denote as M^+ collectively, H_3^+ , molecular ions other than H_3^+ (typically HCO^+), which we denote as m^+ , and charged grains. As the mean masses of M^+ and m^+ we take $34m_{\text{H}}$ and $29m_{\text{H}}$, respectively. We separate H_3^+ from m^+ because the large difference in their masses causes considerable differences in τ_{λ} and ω_{λ} . We adopt the MRN size distribution of grains given by

$$\frac{dn_{\text{g}}}{da} = C n_{\text{H}} a^{-3.5}, \quad a_{\text{min}} \leq a \leq a_{\text{max}}, \quad (27)$$

where a is the grain radius and $C \approx 1.5 \times 10^{-25} \text{ cm}^{2.5}$ (Draine & Lee 1984; Mathis 1986). We take $a_{\text{min}} = 5 \text{ nm}$ and $a_{\text{max}} = 250 \text{ nm}$. As for the charge states of grains we consider neutral, $\pm e$, $\pm 2e$, and $\pm 3e$, where e is the elementary electric charge. We divide the grain radius into 40 bins of equal logarithmic width $\Delta \log a = 0.0425$. This must be accurate enough for our purpose because we have found little difference in numerical results compared with the case of 20 bins adopted by NNU91², though we have found some difference from the case of 5 bins adopted by Desch & Mouschovias (2001).

Figure 1 shows abundances of various particles as functions of the cloud density n_{H} for the *standard* case $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$ (Spitzer & Tomasko 1968). Ions and electrons are dominant

² Desch & Mouschovias (2001) criticized NNU91 in their §2.1 claiming (i) ignorance of the size-dependence of grain charges, and (ii) ignorance of collisions between grains of different sizes which is important to neutralization of grain charges at high densities. In reality, however, NNU91 did not make these ignorances. As for (i) see their Appendix A, where the equations on grain charges have clear dependences on the grain radius. NNU91 did not write anything that means (ii).

charged particles at $n_H \lesssim 10^6 \text{ cm}^{-3}$, ions and g^- (grains of charge $-e$) are dominant at n_H between 10^7 and 10^9 cm^{-3} , and g^- and g^+ are major at $n_H \gtrsim 10^{10} \text{ cm}^{-3}$. To keep electrical neutrality at very high densities $n_H \gtrsim 10^{10} \text{ cm}^{-3}$ where ions and electrons are no longer dominant charged particles because of their efficient recombination, $n(g^-) \approx n(g^+)$ must hold. Nakano (1984) showed that to assure this relation the number flux of thermal electrons must be $(S_i/S_e)^{1/2}$ times that of thermal ions, or

$$\frac{n_e}{n_i} \approx \left(\frac{S_i}{S_e} \frac{m_e}{m_i} \right)^{1/2}, \quad (28)$$

where S_i and S_e are sticking probabilities of ions and electrons, respectively, when they collide neutral grains. Although Nakano (1984) ignored electric polarization of grains induced by approaching charged particles, which has an effect of enhancing the collision rate (Draine & Sutin 1987), this effect cancels out in the derivation of equation (28). As seen in Figure 1, metallic ions are dominant among various ions, or $n_i \approx n(M^+)$, at all densities. In numerical calculation we took $S_e = 0.6$ and $S_i = 1$ (Umebayashi & Nakano 1980; NNU91). This is why $n_e \approx 6 \times 10^{-3} n(M^+)$ holds at $n_H \gtrsim 10^{10} \text{ cm}^{-3}$. Equation (28) also yields $n(g^{--}) \approx n(g^{++})$. Decrease of charged grains with n_H at very high densities is due to neutralization of grains by mutual collisions. Cosmic rays are significantly attenuated at $n_H \gtrsim 10^{12} \text{ cm}^{-3}$.

Figure 2 shows the charge state of grains as a function of the grain radius at several cloud densities. At low densities $n_H \lesssim 10^6 \text{ cm}^{-3}$ grains of charge $-e$ are dominant and 1/4 to 1/30 of grains are neutral depending on their radius. At $n_H \approx 10^8 \text{ cm}^{-3}$ neutral grains are as abundant as $-e$ grains and grains of other charges are much less at all radii. At very high densities $n_H \gtrsim 10^{10} \text{ cm}^{-3}$ neutral grains are most abundant, and $+e$ grains are as abundant as $-e$ grains at all radii.

3.3. Magnetic Flux Loss Time

The mean magnetic force in an oblate cloud with $B \leq B_{\text{cr}}$ satisfies equation (20) whether or not the force balance holds along field lines. The magnetic flux loss time t_B can be given by the time for field lines to drift the length scale L of the cloud. From equations (9) and (20) we have

$$t_B \approx \frac{L}{v_{Bx}} \approx \left(\frac{B_{\text{cr}}}{B} \right)^2 \frac{A_1^2 + A_2^2}{A_1} \frac{2L}{\pi G \Sigma \rho}. \quad (29)$$

First we consider a cloud wherein the force balance approximately holds between gravity and pressure along field lines and whose column density Σ and half-thickness Z are given by equations (25) and (26), respectively. From equations (2) and (25) we have

$$B_{\text{cr}} \approx 4 \left(\frac{\pi k T \rho}{\mu m_H} \right)^{1/2} \approx 6.4 \times 10^{-5} \left(\frac{T}{10 \text{ K}} \frac{n_H}{10^5 \text{ cm}^{-3}} \frac{2.37}{\mu} \right)^{1/2} \text{ G}. \quad (30)$$

The quasistatic contraction of clouds induced by the drift of magnetic fields is highly nonhomologous; only the densest central part of the cloud contracts leaving the outer part almost

unchanged (Nakano 1979, 1982; Lizano & Shu 1989). The time scale of such contraction is the drift time of magnetic fields in the central part of the cloud whose length scale L is nearly equal to the thickness of the cloud, Z , or

$$t_B \approx \left(\frac{B_{\text{cr}}}{B} \right)^2 \frac{A_1^2 + A_2^2}{A_1} \frac{1}{\pi G \rho^2}. \quad (31)$$

Disk-like clouds in runaway collapse with field lines perpendicular to the disk layers have nearly uniform cores whose column density along field lines and size across them are given approximately by equations (25) and (26), respectively (Nakamura et al. 1995). The flux loss time of these cores is also given by equation (31). This equation also holds for nearly spherical clouds wherein the pressure force almost balances with the gravity (though B^2 must be significantly smaller than B_{cr}^2) because the cloud radius is nearly equal to the half-thickness along field lines, Z .

As mentioned above, equation (29) for t_B holds for dynamically contracting clouds even if the force balance does not hold along field lines. How about equation (31)? If the cloud is nearly spherical, equation (31) can also be applied because with $\Sigma/(2L) \approx \rho$ equation (29) is reduced to equation (31). However, equation (2) should be used for B_{cr} instead of equation (30), which holds only for clouds in force balance along field lines.

The time scale t_B given by equation (31) depends on the density and the magnetic field strength of the cloud, but does not depend on the cloud mass. The gas temperature T affects t_B through densities of charged particles, τ_λ , and B_{cr} . We take $T = 10$ K in this paper.

Figure 3 shows t_B for the cases of $B = B_{\text{cr}}$ (solid lines) and $B = 0.1B_{\text{cr}}$ (dashed lines) for clouds in force balance along field lines, or with B_{cr} given by equation (30). The dot-dashed lines show the Ohmic dissipation time t_{od} , which is obtained by taking a limit $B \rightarrow 0$ in equation (31), or by setting $L \approx Z$ in equation (24). A relation $t_B = t_{\text{od}}/(1 + D)$ holds, where D is the excess dissipation relative to Ohmic dissipation given by equation (18). In addition to the *standard case* $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$ (thick lines), we also show the case of $\zeta_0 = 1 \times 10^{-16} \text{ s}^{-1}$ (thin lines) because there are some suggestions from observations of molecular ions that ζ_0 might be significantly larger than the standard value in some clouds (de Boisanger, Helmich, & van Dishoeck 1996; Caselli et al. 1998). At $n_{\text{H}} \lesssim 10^7 \text{ cm}^{-3}$, t_B is 10 to 10^2 times t_{ff} for the case of $B = B_{\text{cr}}$, and t_B for the case of $B = 0.1B_{\text{cr}}$ is about 10^2 times larger than that for $B = B_{\text{cr}}$. At $n_{\text{H}} \gtrsim 10^8 \text{ cm}^{-3}$, the ratio t_B/t_{ff} decreases as the density increases, and at last $t_B = t_{\text{ff}}$ is attained at some density, which we denote n_{dec} and call the decoupling density as in our previous work. For the standard case $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$ we find $n_{\text{dec}} \approx 2 \times 10^{11}$ and $3 \times 10^{11} \text{ cm}^{-3}$ for $B = B_{\text{cr}}$ and $0.1B_{\text{cr}}$, respectively. As shown by NNU91, t_B and n_{dec} do not depend sensitively on various parameters such as fractions of heavy elements remaining in the gas phase and on the details of the grain model (e.g., a_{min}). As seen from Figure 3, t_B and n_{dec} are not very sensitive to ζ_0 .

Ciolek & Mouschovias (1993) criticized our previous work (e.g., NU86; Umebayashi & Nakano 1990; NNU91) in their §1.1 claiming that comparison of t_B with t_{ff} and comparison of v_B with the free-fall velocity u_{ff} were meaningless because magnetically subcritical clouds did not contract

freely. However, because the free-fall time is one of the fundamental time scales of clouds, we can find out by comparing t_B with t_{ff} (or v_B with u_{ff}) whether the magnetic flux is lost effectively in dynamically contracting clouds, and how slowly the clouds in quasi-equilibrium contract induced by the drift of magnetic fields without detailed simulations which have been done by many authors (e.g., Nakano 1979; Lizano & Shu 1989; Ciolek & Mouschovias 1994). According to Ciolek & Mouschovias (1993), we concluded erroneously that ambipolar diffusion (or drift of magnetic fields) was inefficient at $n_{\text{H}} \ll n_{\text{dec}}$ because $t_B \gg t_{\text{ff}}$. Our conclusion in the previous and this papers is that extensive flux loss occurs only at $n_{\text{H}} \gtrsim n_{\text{dec}}$. If there exist highly magnetically subcritical clouds with $\Phi \gg \Phi_{\text{cr}}$, they might lose magnetic fluxes extensively even at $n_{\text{H}} \ll n_{\text{dec}}$ contracting quasistatically, though only down to $\approx \Phi_{\text{cr}}$, as shown by numerical simulations of Ciolek & Mouschovias (1994). However, clouds or cloud cores with $\Phi \gg \Phi_{\text{cr}}$ cannot exist (Nakano 1998). Moreover, as mentioned in § 1, cloud cores must decrease their magnetic fluxes down to $10^{-3} \Phi_{\text{cr}}$ or below in the process of star formation. Cloud cores with Φ somewhat smaller than Φ_{cr} can begin dynamical contraction rather easily (Nakano 1998). Extensive flux loss down to $\Phi \ll \Phi_{\text{cr}}$ does not occur at $n_{\text{H}} \ll n_{\text{dec}}$ because t_B is much larger than the dynamical time scale.

3.4. Dependence on Magnetic Field Strength

We try to approximate the dependence of t_B on the field strength B by a power law $t_B \propto B^{-\gamma}$. Figure 4 (*top*) shows the power index γ obtained by comparing t_B for the two cases $B = B_{\text{cr}}$ and $0.1B_{\text{cr}}$ for the standard case $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$. At $n_{\text{H}} \lesssim 10^7 \text{ cm}^{-3}$, deviation of γ from 2 is small, or $t_B \propto B^{-2}$ approximately holds, at least for $B_{\text{cr}} \geq B \gtrsim 0.1B_{\text{cr}}$, in good agreement with ambipolar diffusion described in § 2.2. This is because ions and smallest grains, which are major contributors to transmitting the magnetic force to the neutrals, are relatively well frozen to magnetic fields in this density range. We shall discuss the small deviation of γ from 2 in § 3.5.

At $n_{\text{H}} > 10^7 \text{ cm}^{-3}$, γ decreases steeply as the density increases, and finally settles down to ≈ 0 . The situation $\gamma \ll 1$ suggests that Ohmic dissipation contributes at least significantly to magnetic flux loss. However, although $\gamma = 0.13$ at $n_{\text{H}} = 2 \times 10^{11} \text{ cm}^{-3}$, the flux loss occurs much faster than by Ohmic dissipation; $t_B \approx 0.1t_{\text{od}}$ as shown in Figure 3, or $D \approx 10$. The steep decrease of γ with density and this behavior of magnetic flux loss at $n_{\text{H}} \sim n_{\text{dec}}$ can be understood by checking the motion of some typical charged particles.

Figure 4 (*bottom*) shows the drift velocity relative to that of magnetic fields, v_i/v_B , given by equation (13), and $\tau_i\omega_i$ of the dominant ions M^+ for the two cases $B = B_{\text{cr}}$ (solid line) and $0.1B_{\text{cr}}$ (dashed line) with $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$. Here we omit the subscript x to the velocities. At $n_{\text{H}} = 1 \times 10^9 \text{ cm}^{-3}$, for example, metallic ions drift almost with magnetic fields with $v_i/v_B \approx 0.96$ for $B = B_{\text{cr}}$ because they are well frozen to magnetic fields with $\tau_i\omega_i \approx 32$. For $B = 0.1B_{\text{cr}}$, however, we find $v_i/v_B \approx 0.26$ and $\tau_i\omega_i \approx 3.2$. This value of v_i/v_B is much smaller than a naive estimation $v_i/v_B \approx (\tau_i\omega_i)^2/[1 + (\tau_i\omega_i)^2] \approx 0.91$ for this value of $\tau_i\omega_i$. This large deviation is caused by the second term in the parentheses of equation (13), or by interaction with other charged

particles, especially grains. Even for $B = B_{\text{cr}}$ deviation of v_i from v_B is caused mostly by this term: a naive estimation gives $v_i/v_B \approx (\tau_i \omega_i)^2/[1 + (\tau_i \omega_i)^2] \approx 0.999$. As B decreases, v_i/v_B decreases markedly.

The situation is the same for grains because $|\tau_g \omega_g| \ll \tau_i \omega_i$. Figure 5 shows the drift velocity relative to that of magnetic fields, v_g/v_B , and $|\tau_g \omega_g|$ of grains of charge $-e$ as functions of the grain radius at several densities for $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$. At $n_H = 10^9 \text{ cm}^{-3}$ and $B = B_{\text{cr}}$, grains of $a = 10 \text{ nm}$ with electric charge $-e$ and $+e$, for example, have relative drift velocities $v_g/v_B = 0.41$ and -0.22 , respectively. These particles have $|\tau_g \omega_g| = 0.32$, and then the naive estimation gives $v_g/v_B \approx (\tau_g \omega_g)^2/[1 + (\tau_g \omega_g)^2] \approx 0.093$ for both charges, much smaller than the actual values; the actual velocity of $+e$ grains has even an opposite sign. As B decreases, $|v_g/v_B|$ decreases markedly for all radii as shown in Figure 5.

At $n_H > 10^7 \text{ cm}^{-3}$ ions are not very well frozen to magnetic fields even for $B \approx B_{\text{cr}}$. Therefore, as B decreases, the degree of freezing and v_i/v_B decrease markedly. The situation is the same for grains which are much less tightly coupled with magnetic fields. Decrease of v_i/v_B and $|v_g/v_B|$ has an effect of decreasing t_B if v_i and v_g are fixed. On the other hand, as B decreases, the magnetic force, or the driving force of the drift, decreases, which has an effect of decreasing the drift velocity v_i and $|v_g|$ as seen from equation (11), and then has an effect of increasing t_B if v_i/v_B and v_g/v_B are fixed. These opposite effects make t_B less sensitive to B than at $n_H < 10^7 \text{ cm}^{-3}$, where ions and smallest grains, the dominant charged particles, are relatively well frozen and the decrease of B has little effect on their v_λ/v_B . As the density increases, t_B becomes more and more insensitive to B because $|\tau_\lambda \omega_\lambda|$ decreases with density for all charged particles even though t_B is significantly smaller than t_{od} . This is why γ decreases steeply with density at $n_H > 1 \times 10^7 \text{ cm}^{-3}$.

3.5. Contribution of Grains

Even at $n_H < 1 \times 10^7 \text{ cm}^{-3}$, γ shows some deviation from 2 as seen in Figure 4 (*top*). Because ions are strongly coupled with magnetic fields and deviation of v_i from v_B is very small at these densities as shown in Figure 4 (*bottom*), this deviation suggests that transmission of the magnetic force to the neutrals is significantly contributed by grains, which are not very strongly coupled with magnetic fields even at these low densities as shown in Figure 5.

To confirm this we calculate the frictional force exerted by each kind of charged particles on the neutrals, which is given by each term of equation (11). We shall not consider the component of the frictional force perpendicular to both \mathbf{B} and $\mathbf{j} \times \mathbf{B}$ because summation of this component for all charged particles vanishes as shown by equation (12). Figure 6 shows the frictional forces of ions and grains relative to the total frictional force, which is equal to $|\mathbf{j} \times \mathbf{B}|/c$ per unit volume, as functions of the cloud density for the two cases of the field strength with $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$. Ions contribute more than grains only at $n_H \lesssim 10^4 \text{ cm}^{-3}$, where the ionization fraction is relatively high. At $n_H \gtrsim 10^6 \text{ cm}^{-3}$ grains contribute more than about 90% of the frictional force at

least for $B_{\text{cr}} \geq B \gtrsim 0.1B_{\text{cr}}$. The frictional force exerted by electrons is more than 3 orders of magnitude smaller than that by ions mainly because of their small drift momentum. For the case of $\zeta_0 = 1 \times 10^{-16} \text{ s}^{-1}$ these densities are somewhat higher; e.g., ions contribute more than grains at $n_{\text{H}} \lesssim 10^5 \text{ cm}^{-3}$ because of higher ion densities.

Figures 7 and 8 show the frictional force exerted by grains on the neutrals as a function of their radius a and charge q_{g} at several cloud densities for the two cases of the field strength $B = B_{\text{cr}}$ and $0.1B_{\text{cr}}$ with $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$. As well as ions, negatively charged grains exert frictional force, or drift, in the direction of the magnetic force irrespective of their radius at $n_{\text{H}} \lesssim 10^{14}(B/B_{\text{cr}})^2 \text{ cm}^{-3}$ at least for $B \gtrsim 0.1B_{\text{cr}}$. Positively charged grains behave differently. At low densities they contribute in the same direction as negatively charged grains irrespective of their radius. However, at some density the largest grains begin to contribute, or drift, in the opposite direction, and the size range of such grains expands to smaller radii as n_{H} increases; frictional forces in these ranges are shown by dotted lines in Figures 7 and 8. For example, grains of charge $+e$ with $a > 160 \text{ nm}$ drift in the opposite direction at $n_{\text{H}} = 10^5 \text{ cm}^{-3}$ for $B = B_{\text{cr}}$, and those with $a > 93 \text{ nm}$ do at $n_{\text{H}} = 10^4 \text{ cm}^{-3}$ for $B = 0.1B_{\text{cr}}$ though the dotted line is outside this panel. Finally at $n_{\text{H}} \approx 2 \times 10^9$ and $9 \times 10^6 \text{ cm}^{-3}$ for $B = B_{\text{cr}}$ and $0.1B_{\text{cr}}$, respectively, even the smallest grains ($a \approx 5 \text{ nm}$) with charge $+e$ begin to drift opposite to the magnetic force. These are because $A_2 < 0$ (or $\sigma_{\text{H}} < 0$) in most of the density region covered by this paper (see Figure 9; $A_1 > 0$ by definition), and thus positively charged particles with small $\tau_{\lambda}\omega_{\lambda}$ have negative drift velocities as seen from equation (13). Because $\tau_{\text{g}}\omega_{\text{g}} \propto a^{-2}n_{\text{H}}^{-1/2}(B/B_{\text{cr}})$, the radius range in which positively charged grains have negative drift velocities expands to smaller a as n_{H} increases for a fixed B/B_{cr} as long as $A_2 < 0$. We shall discuss the physical reason of this phenomenon in § 4.1.

At high densities, not only t_B deviates greatly from the $t_B \propto B^{-2}$ relation, but also grains, the main contributors to the frictional force, of opposite charges drift in opposite directions. This contradicts the literal meaning of ambipolar diffusion, a term originally used in plasma physics for a quite different phenomenon in which particles of opposite charges diffuse as one unit (e.g., Cap 1976; see also Appendix C).

The great contribution of grains to the frictional force comes from their large contribution to A_1 and A_2 , or σ_{P} and σ_{H} , though σ_{\parallel} is mainly contributed by electrons in most of the density region in this paper. This difference in the contribution is due to large differences in $|\tau_{\lambda}\omega_{\lambda}|$ and in the abundance n_{λ}/n_{H} among charged particles. For metallic ions we have $\tau_{\text{i}}\omega_{\text{i}} \approx 320(n_{\text{H}}/10^7 \text{ cm}^{-3})^{-1/2}(B/B_{\text{cr}})$. At $T \approx 10 \text{ K}$ we find $|\tau_{\text{e}}\omega_{\text{e}}|/\tau_{\text{i}}\omega_{\text{i}} \approx 4.8 \times 10^3$ and $|\tau_{\text{g}}\omega_{\text{g}}|/\tau_{\text{i}}\omega_{\text{i}} \approx 1.0 \times 10^{-2}(a/10 \text{ nm})^{-2}$ for grains of charge $q_{\text{g}} = \pm e$. For the case of $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$, σ_{\parallel} is mainly contributed by electrons at $n_{\text{H}} \lesssim 10^{13} \text{ cm}^{-3}$ and by grains at higher densities as shown in Figure 9. Contribution of ions to σ_{\parallel} is minor at all densities. To σ_{P} or A_1 , ions contribute more than half only at $n_{\text{H}} \lesssim 10^4 \text{ cm}^{-3}$, and grains contribute more than 80% at $n_{\text{H}} \gtrsim 10^5 \text{ cm}^{-3}$ for $B = B_{\text{cr}}$ (Figure 9), though for $B = 0.1B_{\text{cr}}$ ions also contribute 50–80% at n_{H} between 3×10^8 and $6 \times 10^{10} \text{ cm}^{-3}$. At $n_{\text{H}} \gtrsim 10^{11} \text{ cm}^{-3}$, grains overwhelm the other particles. Contribution of electrons to σ_{P} is less than 10^{-3} because of large $|\tau_{\text{e}}\omega_{\text{e}}|$ at low densities and because of low abundance at

high densities [see equation (6) or (A5)]. Situation is complicated for σ_H or A_2 . Because the terms of some given particles have quite different forms even with opposite signs between the equivalent equations (7) and (A6), we cannot uniquely tell how much each kind of particles contributes to σ_H . Moreover, particles of opposite charges contribute oppositely and significant cancellation occurs in some situations for either expression of σ_H . We would make a serious mistake if we neglect the terms of grains in σ_H except at high densities where both $\tau_i \omega_i \lesssim 10$ and $n_i \gg n_e$ hold.

The sign of σ_H changes at some density as shown in Figure 9. With an expression of σ_H accurate at very high densities, we find that this occurs when $\tau_i \omega_i \approx (n_e/n_i)^{1/2}$. As long as $B \gtrsim 0.01 B_{\text{cr}}$, this gives $\tau_i \omega_i \approx 0.08$, which corresponds to the density $n_H \approx 1.6 \times 10^{14} (B/B_{\text{cr}})^2 \text{ cm}^{-3}$. Because of the change of the sign, grains change the directions of drift at densities somewhat higher than this as seen from equation (13). However, the total frictional force they exert on the neutrals is always in the direction of magnetic force because the mean drift velocity of $-e$ and $+e$ grains is in the direction of magnetic force as seen from equation (3) or (13).

Desch & Mouschovias (2001) ignored the contribution of grains to σ_P (σ_\perp by their notation) and σ_H as seen from their equations (25) and (26). Therefore, their equation (28), which was obtained by using these equations, does not correspond to the critical state at which Ohmic dissipation becomes important, or $D \approx 1$, contrary to their statement. Their results will be discussed as compared with ours in §4.3.

Because the frictional force is contributed mainly by grains, t_B is determined mainly by grains except at very low densities. Besides, the reactions at grain surface are important in determining the densities of various charged particles (e.g., Nakano 1984; NNU91). Thus, grains play a decisive role in the process of magnetic flux loss in molecular clouds.

4. Discussion

4.1. Drift of Grains

Grains of opposite electric charges drift in opposite directions at high densities as shown in §§3.4 and 3.5. This is formally a result of the second term in the parentheses of equation (13). Here we shall give a more intuitive explanation of this phenomenon.

In the existence of an electric field \mathbf{E} a particle of electric charge $q_\lambda \neq 0$ gyrating around magnetic field lines³ drifts with a velocity $\mathbf{v}_\lambda = c \mathbf{E} \times \mathbf{B}/B^2$ independent of its charge. If there is a force field \mathbf{f} , which is independent of the particle charge, instead of the electric field, the particle

³ One may think that gyration completely dissipates because the viscous damping times τ_λ are smaller than t_B and t_H by orders of magnitude. However, because of collisions with neutral molecules and atoms charged particles always have thermal motions, which would cause gyration, though a term "gyrating" may not be appropriate when $|\tau_\lambda \omega_\lambda| \lesssim 1$.

drifts with a velocity $\mathbf{v}_\lambda = (c/q_\lambda) \mathbf{f} \times \mathbf{B}/B^2$ dependent on its charge. In the frame moving with the neutrals in a molecular cloud, there is an electric field satisfying equation (8), which causes a charge-independent drift motion (call this drift 1). Neutral molecules exert a frictional force on this drift motion, which is anti-parallel to the drift velocity independent of the electric charge. Therefore, this force causes another drift motion (drift 2) whose direction depends on the particle charge. Thus the drift velocity has both components dependent on and independent of the particle charge. Because drift 2 has opposite directions for particles of opposite charges, the frictional force on this motion yields the third drift motion whose direction is charge-independent, etc.

All these can be taken into account in the equation for the steady motion (A1) in Appendix A. The direct solution of this equation is given by equations (A2) and (A3), whose components perpendicular to \mathbf{B} are found to agree with equations (3) and (4). When $|\tau_\lambda \omega_\lambda| \lesssim 1$, the frictional force can be comparable to or stronger than the electric force, and therefore the charge-dependent part of the drift velocity can be comparable to or greater than the charge-independent one.

Because grains of opposite charges drift in opposite directions at high densities where grains are the major charged particles, one may anticipate that charge separation occurs in the $\mathbf{j} \times \mathbf{B}$ -, or x -direction. However, this is not the case. All charged particles should be taken into account. As confirmed in § 2.1, equation (3) guarantees that the x -component of the electric current density vanishes.

Although the frictional force is contributed mostly by grains at high densities, their drift velocities are very small. For example, at the decoupling density $n_H \approx n_{\text{dec}} \approx 2 \times 10^{11} \text{ cm}^{-3}$ for $B = B_{\text{cr}}$ with $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$ even the smallest grains of $a \approx 5 \text{ nm}$ with electric charge $-e$ have a drift velocity as small as $v_g/v_B \approx 0.02$ as seen from Figure 5. Because extensive magnetic flux loss occurs only at $n_H \gtrsim n_{\text{dec}}$ and decrease of B/B_{cr} by the flux loss makes v_g/v_B even smaller (Figure 5), grains are hardly lost from the cloud core even if the magnetic flux decreases by orders of magnitude as far as the cloud core is magnetically supercritical. Although Ciolek & Mouschovias (1994) write that the grain abundance decreases almost in proportion to the reduction factor of the magnetic flux in highly magnetically subcritical cloud cores, cloud cores can hardly be magnetically subcritical as discussed by Nakano (1998). Although the drift velocity of ions is not very small compared with that of magnetic fields even at $n_H \approx n_{\text{dec}}$ for $B \approx B_{\text{cr}}$ (Figure 4), ions are quite minor constituents among the heavy elements even in the gas phase (Figure 1). Thus magnetic flux loss in cloud cores has little effect on the abundance of heavy elements in stars born therein.

4.2. Charge Fluctuation of Grains

Electric charges of grains change stochastically because ions and electrons sometimes stick to them. This change has some effect on their motion in electromagnetic fields. Nakano (1984) and NU86 neglected this effect because they had a rough estimation that it was small (see also Nakano

& Umebayashi 1980).

Kamaya & Nishi (2000) investigated this effect more elaborately for grains of charge $-e$. The probability of having some charge is proportional to the time span during which the grain has this charge. Therefore, as seen from Figure 2, the grains of charge $-e$ change their charge mostly by going to the neutral state at most radii a and at most densities n_{H} . The charge change between the $-e$ and neutral states effectively decreases the viscous damping time τ_{g} of grains of charge $-e$ by some factor $C_{\text{g}} > 1$. This effect can be taken into account by replacing τ_{g} with $\tau_{\text{g}}/C_{\text{g}}$ in our formalism summarized in §2. Kamaya & Nishi found that if H_3^+ is the dominant ion, C_{g} for grains of $a = 10 \text{ nm}$ is 1.3 at $n_{\text{H}} \approx 10^3 \text{ cm}^{-3}$ and approaches 1 as n_{H} increases, and that larger grains have C_{g} closer to 1.

Because metallic ions and molecular ions other than H_3^+ are the dominant ions as shown in Figure 1, deviations of C_{g} from 1 are about 3 times smaller than those obtained for H_3^+ by Kamaya & Nishi (2000) at all densities and at all grain radii; the deviation $C_{\text{g}} - 1$ is proportional to $m_{\text{i}}^{-1/2}$, where m_{i} is the mass of the dominant ions. Therefore, charge fluctuation of grains can hardly affect t_{B} because C_{g} can be as large as 1.1 only for smallest grains only at low densities $n_{\text{H}} \sim 10^3 \text{ cm}^{-3}$, where grains are minor contributors to the frictional force. Thus, our formalism can be applied to any situation with little error though Ciolek & Mouschovias (1993) pointed out our ignorance of this effect.

4.3. Comparison with Other Work

We compare our results with several previous works on t_{B} , n_{dec} , and the dissipation in excess of Ohmic dissipation given by D in equation (18).

The decoupling densities n_{dec} obtained in §3.3 are about 4 times larger than that of NNU91 for the same grain model. The main cause for this difference is in B_{cr} ; our B_{cr} is $\sqrt{3}$ times larger than theirs. At relatively low densities where major charged particles are well frozen to magnetic fields, difference in the coefficient of B_{cr} has little effect on t_{B} for a given B/B_{cr} because these particles drift almost with magnetic fields and the drift velocity v_{Bx} is determined by the balance of the frictional force on them with $(B/B_{\text{cr}})^2$ times the gravity as seen from equations (19) and (20). At high densities, t_{B} , and then n_{dec} , depend on the coefficient of B_{cr} because the delay in the drift of major charged particles, which are not well frozen, depends on the field strength as discussed in §3.4. However, the general feature of t_{B} (rough dependences on n_{H} and B/B_{cr} , existence of n_{dec} , etc.) is hardly affected by this coefficient.

NU86 investigated magnetic flux loss in spherical clouds, and found that magnetic decoupling occurs at $n_{\text{H}} \approx n_{\text{dec}} \approx 5 \times 10^{11} \text{ cm}^{-3}$ for a cloud of $1M_{\odot}$ and the decoupling density does not depend sensitively on the cloud mass. They also found that dissipation in excess of Ohmic one is not large, or $D \sim 1$, at $n_{\text{H}} \approx n_{\text{dec}}$ for $B \approx B_{\text{cr}}$.

In §3.4 we have found $D \approx 10$ at $n_H \approx n_{\text{dec}}$, which is an order of magnitude larger than D obtained by NU86. We can list up some causes for this discrepancy; differences in the cloud model and in the grain model. NU86 assumed that clouds were spherical without force balance along field lines. They also assumed for simplicity that all grains had the same radius $a = 100$ nm. Another difference is in the electric conductivity σ_{\parallel} . For the collision of electrons with the neutrals, NU86 adopted the classical Langevin’s cross sections in which electric polarization of the neutrals induced by an approaching electron has a great effect. These were widely adopted formerly (e.g., Nakano 1984; Mouschovias 1991) assuming the similarity with the collision of ions with the neutrals. However, we found that laboratory experiments show much smaller cross sections at low energies (Hayashi 1981), which are not much different from the geometrical cross sections. Sano et al. (2000) give the empirical formulae of the momentum transfer rate coefficients for the collision of electrons with H_2 molecules and with He atoms, which were obtained by Umebayashi (1993, private communication) by fitting to the laboratory data. With these collision rate coefficients, which are about 80 times smaller than the previous ones at $T \approx 10$ K, the conductivity σ_{\parallel} , t_{od} , and D are much larger than the previous ones except at $n_H \gtrsim 10^{13} \text{ cm}^{-3}$ where σ_{\parallel} is mainly contributed by grains. On the other hand, σ_P is hardly affected by this change of the cross sections because contribution of electrons is very small even with the enhanced σ_e (Figure 9), and σ_H is also hardly affected except at very high densities where $|\sigma_H| \ll \sigma_P$. Therefore, $t_B = t_{\text{od}}/(1 + D)$ is hardly affected by this change because $t_{\text{od}} \propto \sigma_{\parallel}$ and $1 + D = \sigma_{\parallel}\sigma_P/(\sigma_P^2 + \sigma_H^2)$.

Desch & Mouschovias (2001) write that Ohmic dissipation becomes important, or $D \approx 1$ is realized, when $|\tau_e\omega_e| \approx 6$. However, $|\tau_e\omega_e| \approx 6$ was obtained from their equation (28), $\tau_1\omega_1|\tau_e\omega_e| \approx n_e/n_i$,⁴ which does not correspond to the state of $D \approx 1$ because they ignored the contribution of grains to σ_P and σ_H (see §3.5). Their criterion $|\tau_e\omega_e| \approx 6$ for $D \approx 1$ gives a density $n_H \approx 6 \times 10^{17} (B/B_{\text{cr}})^2 \text{ cm}^{-3}$ irrespective of ζ_0 when B_{cr} is given by equation (30), or the pressure force balances with the gravity along field lines. When contraction along field lines is insufficient and then the pressure force along field lines is weaker than the gravity, Σ is larger than given by equation (25) and B_{cr} is larger than equation (30) for a given density. This means that $|\tau_e\omega_e|$ is larger for given n_H and B/B_{cr} , and therefore $|\tau_e\omega_e| \approx 6$ gives even higher densities than in the above case.

Taking into account the contribution of grains to σ_P and σ_H , we have found that $D \approx 1$, or $t_B \approx t_{\text{od}}/2$, is realized at $n_H \approx 1 \times 10^{13} \text{ cm}^{-3}$ for $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$ almost independent of B/B_{cr}

⁴ Emphasizing the contrast with this equation, Desch & Mouschovias (2001) write that NU86 used a relation $\tau_1\omega_1|\tau_e\omega_e| \approx 1$ for $D \approx 1$ assuming $n_e = n_i$ even at densities as high as $n_H \gtrsim n_{\text{dec}}$. This is another misrepresentation of the fact in addition to their comments on NNU91 cited in footnote 2. Although NU86 gave the expressions of D , by their equations (49) and (50), when ions and electrons are major contributors to A_1 and A_2 (or σ_P and σ_H) (of course $n_e \approx n_i$), and when grains of charge $-e$ and $+e$ are major contributors, respectively, as limiting cases, they used the general expression of D given by their equation (48) or ours (18) in numerical calculation. For instance, to calculate D shown in their figure 2, they used the densities of charged particles shown in their figure 3, which clearly shows the deviation of n_e from n_i at high densities in agreement with our equation (28).

at least at $0.1 \lesssim B/B_{\text{cr}} \leq 1$. This can be confirmed by substituting the values of σ_{\parallel} , σ_{P} , and σ_{H} shown in Figure 9 into equation (18) and by noting that t_B is almost independent of B/B_{cr} at high densities as shown in Figure 3. This density is lower than Desch & Mouschovias’ values by orders of magnitude.

In reality, however, Ohmic dissipation becomes important at significantly lower densities. At $n_{\text{H}} \gtrsim n_{\text{dec}}$ the clouds contract dynamically adjusting B/B_{cr} so as to keep $t_B \approx t_{\text{ff}}$ because too much decrease of B/B_{cr} makes t_B larger than t_{ff} as long as $t_{\text{od}} > t_{\text{ff}}$. However, $t_B = t_{\text{od}}/(1 + D)$ cannot be larger than t_{od} . Therefore, after $t_{\text{od}} \approx t_{\text{ff}}$ is attained at $n_{\text{H}} \approx 1 \times 10^{12} \text{ cm}^{-3}$ (Figure 3), field dissipation proceeds by Ohmic dissipation faster than the contraction.

Kamaya & Nishi (2000) investigated the motion of ions and grains in the process of magnetic flux loss using a simplified model. Assuming that all grains have the same radius and have an electric charge $-e$ or 0, ions are well frozen to magnetic fields ($\tau_i \omega_i \gg 1$), and electrons are completely frozen ($|\tau_e \omega_e| \rightarrow \infty$), they obtained the drift velocities of ions and grains of charge $-e$. If we apply equation (3) to their model, we easily obtain the drift velocities given by their equations (34) and (35) though τ_g should be replaced by τ_g/C_g . Thus, Kamaya & Nishi (2000) obtained some relatively transparent results due to their simplified model, which are consistent with our formalism, though they got some results different from Ciolek & Mouschovias (1993).

5. Summary

We analyzed the detailed processes operating in the drift of magnetic fields in molecular clouds taking into account charged grains with the MRN size distribution in addition to ions and electrons and the effect of partial freezing of these particles to magnetic fields using the formalism obtained by Nakano (1984) and NU86.

We found that to the frictional force, whereby the magnetic force is transmitted to neutral molecules, ions contribute more than half only at cloud densities $n_{\text{H}} \lesssim 10^4 \text{ cm}^{-3}$, and grains contribute more than about 90% at $n_{\text{H}} \gtrsim 10^6 \text{ cm}^{-3}$. Besides, the reactions at grain surface are important in determining the densities of various charged particles. Thus grains play a decisive role in the process of magnetic flux loss in molecular clouds.

We confirmed the previous results (NNU91) on the magnetic flux loss time t_B and the decoupling density n_{dec} , at which t_B is equal to the free-fall time t_{ff} and therefore magnetic fields are effectively decoupled from the gas; e.g., $t_B \gg t_{\text{ff}}$ at $n_{\text{H}} \ll n_{\text{dec}}$, and n_{dec} is almost independent of the magnetic field strength in the cloud and takes a value a few $\times 10^{11} \text{ cm}^{-3}$. We found that t_B and n_{dec} are not very sensitive to the ionization rate by cosmic rays, ζ_0 .

We investigated the dependence of t_B on the field strength B . Approximating the relation by a power law $t_B \propto B^{-\gamma}$, we found $\gamma \approx 2$, characteristic to ambipolar diffusion, only at $n_{\text{H}} \lesssim 10^7 \text{ cm}^{-3}$ where ions and smallest grains are pretty well frozen to magnetic fields. At

$n_{\text{H}} > 10^7 \text{ cm}^{-3}$, γ decreases steeply as n_{H} increases, and finally at $n_{\text{H}} \approx n_{\text{dec}}$, $\gamma \ll 1$ is attained, reminiscent of Ohmic dissipation, though the flux loss occurs about 10 times faster than by Ohmic dissipation at $n_{\text{H}} \approx \text{a few} \times 10^{11} \text{ cm}^{-3}$. Because even ions are not very well frozen to magnetic fields at $n_{\text{H}} > 10^7 \text{ cm}^{-3}$, ions and charged grains drift slower than the magnetic fields. Decrease of B has an effect of increasing this lag. This insufficient freezing makes t_B less sensitive to B than at $n_{\text{H}} \lesssim 10^7 \text{ cm}^{-3}$ where major charged particles are well frozen and the decrease of B has little effect on their lag. This tendency is enhanced as the density increases, and at last t_B becomes almost independent of B at $n_{\text{H}} \approx n_{\text{dec}}$. Ohmic dissipation is dominant only at $n_{\text{H}} \gtrsim 10^{12} \text{ cm}^{-3}$.

We found that while ions and electrons drift in the direction of magnetic force at all densities, grains of opposite charges drift in opposite directions at high densities, where grains are major contributors to the frictional force, apart from the component of the drift velocities perpendicular to the magnetic force which yields no net frictional force. Although magnetic flux loss occurs significantly faster than by Ohmic dissipation even at $n_{\text{H}} \approx n_{\text{dec}}$, the operating process at high densities is quite different from ambipolar diffusion in which particles of opposite charges are supposed to drift as one unit.

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A. Another Method of Formulation

Because the procedures of obtaining the formulae summarized in § 2.1 are rather complicated, it would be worthwhile to show another method of obtaining them.

With the same approximation as adopted by Nakano (1984) and NU86 and described in § 2.1, the mean motion of charged particle λ obeys

$$q_{\lambda} \left(\mathbf{E} + \frac{1}{c} \mathbf{v}_{\lambda} \times \mathbf{B} \right) - \frac{m_{\lambda} \mathbf{v}_{\lambda}}{\tau_{\lambda}} = 0, \quad (\text{A1})$$

in the frame moving with the neutrals (see § 2.1 for the notation). The solution of this equation is given by

$$\mathbf{v}_{\lambda\parallel} = \frac{q_{\lambda} \tau_{\lambda}}{m_{\lambda}} \mathbf{E}_{\parallel}, \quad (\text{A2})$$

$$\mathbf{v}_{\lambda\perp} = \frac{c}{B} \frac{(\tau_{\lambda} \omega_{\lambda})^2}{1 + (\tau_{\lambda} \omega_{\lambda})^2} \left(\frac{\mathbf{E}_{\perp}}{\tau_{\lambda} \omega_{\lambda}} + \mathbf{E}_{\perp} \times \frac{\mathbf{B}}{B} \right), \quad (\text{A3})$$

where subscripts \parallel and \perp represent the components parallel and perpendicular to \mathbf{B} , respectively.

The electric current density \mathbf{j} is obtained by summing up $n_{\lambda} q_{\lambda} \mathbf{v}_{\lambda}$ for all kinds of charged particles using equations (A2) and (A3). Moving to a more general frame wherein the neutrals

move with a velocity \mathbf{u}_n and the electric field is given by $\mathbf{E}_0 = \mathbf{E} - \mathbf{u}_n \times \mathbf{B}/c$, and using the electrical neutrality relation, we obtain

$$\mathbf{j} = \sigma_{\parallel} \mathbf{E}_{0\parallel} + \sigma_P \left(\mathbf{E}_{0\perp} + \frac{1}{c} \mathbf{u}_n \times \mathbf{B} \right) + \sigma_H \frac{\mathbf{B}}{B} \times \left(\mathbf{E}_{0\perp} + \frac{1}{c} \mathbf{u}_n \times \mathbf{B} \right), \quad (\text{A4})$$

where σ_{\parallel} is the electric conductivity along magnetic field lines given by equation (15), and σ_P and σ_H are Pedersen and Hall conductivities, respectively, given by

$$\sigma_P = \sum_{\nu} \frac{\sigma_{\nu}}{1 + (\tau_{\nu} \omega_{\nu})^2} = \left(\frac{c}{B} \right)^2 A_1, \quad (\text{A5})$$

$$\sigma_H = - \sum_{\nu} \frac{\sigma_{\nu} \tau_{\nu} \omega_{\nu}}{1 + (\tau_{\nu} \omega_{\nu})^2} = \left(\frac{c}{B} \right)^2 A_2, \quad (\text{A6})$$

where A_1 , A_2 , and σ_{ν} are given by equations (6), (7), and (15), respectively. The last equality of equation (A6) can be found by using the electrical neutrality relation. Equation (A4) with (15), (A5), and (A6) is a generalization of, e.g., Parks (1991) who considered only electrons and a single kind of ions as charged particles.

The drift velocity of magnetic fields, \mathbf{v}_B , satisfies equation (8). Making a vector product of equation (A4) with \mathbf{B} and eliminating $\mathbf{E}_0 + \mathbf{u}_n \times \mathbf{B}/c = \mathbf{E}$ by using equation (8), we obtain

$$\frac{1}{c} \mathbf{j} \times \mathbf{B} = \left(\frac{B}{c} \right)^2 \left(\sigma_P \mathbf{v}_B + \sigma_H \frac{\mathbf{B}}{B} \times \mathbf{v}_B \right). \quad (\text{A7})$$

This equation is solved for \mathbf{v}_B as

$$\mathbf{v}_B = \left(\frac{c}{B} \right)^2 \left[\frac{\sigma_P}{\sigma_P^2 + \sigma_H^2} \frac{1}{c} \mathbf{j} \times \mathbf{B} + \frac{\sigma_H}{\sigma_P^2 + \sigma_H^2} \frac{1}{c} (\mathbf{j} \times \mathbf{B}) \times \frac{\mathbf{B}}{B} \right]. \quad (\text{A8})$$

By taking the components of this equation we can confirm that equation (A8) is identical with equations (9) and (10).

Eliminating \mathbf{E}_{\perp} in equation (A3) by using equation (8), we obtain

$$\mathbf{v}_{\lambda\perp} = \frac{(\tau_{\lambda} \omega_{\lambda})^2}{1 + (\tau_{\lambda} \omega_{\lambda})^2} \left(\mathbf{v}_B + \frac{1}{\tau_{\lambda} \omega_{\lambda}} \frac{\mathbf{B}}{B} \times \mathbf{v}_B \right). \quad (\text{A9})$$

Elimination of \mathbf{v}_B by using equation (A8) gives

$$\mathbf{v}_{\lambda\perp} = \frac{(\tau_{\lambda} \omega_{\lambda})^2}{1 + (\tau_{\lambda} \omega_{\lambda})^2} \left(\frac{c}{B} \right)^2 \frac{1}{\sigma_P^2 + \sigma_H^2} \left[\left(\sigma_P + \frac{\sigma_H}{\tau_{\lambda} \omega_{\lambda}} \right) \frac{1}{c} \mathbf{j} \times \mathbf{B} + \left(\frac{\sigma_P}{\tau_{\lambda} \omega_{\lambda}} - \sigma_H \right) \frac{\mathbf{B}}{B} \times \frac{1}{c} (\mathbf{j} \times \mathbf{B}) \right]. \quad (\text{A10})$$

It is easy to confirm that equation (A10) is identical with equations (3) and (4).

With some manipulation equation (A4) can be solved for \mathbf{E}_0 as

$$\mathbf{E}_0 = -\frac{1}{c} \mathbf{u}_n \times \mathbf{B} + \frac{1}{\sigma_{\parallel}} \mathbf{j} + \frac{\sigma_H}{\sigma_P^2 + \sigma_H^2} \mathbf{j} \times \frac{\mathbf{B}}{B} + \left(\frac{\sigma_P}{\sigma_P^2 + \sigma_H^2} - \frac{1}{\sigma_{\parallel}} \right) \frac{\mathbf{B}}{B} \times \left(\mathbf{j} \times \frac{\mathbf{B}}{B} \right). \quad (\text{A11})$$

By comparing the coefficients we can confirm that equations (14) and (A11) are identical. Desch & Mouschovias (2001) used an equation equivalent to ours (14) or (A11) though neglecting the dominant terms of σ_P and σ_H as pointed out in §3.5.

B. Effect of Collision between Charged Particles

In §2 and Appendix A we have neglected frictional forces on charged particles except those exerted by the neutrals. Except for electrons in some limited situations, this is a good approximation. We discuss the effect of collision between electrons and other charged particles on the collision time τ_e of electrons, which appears in the electric conductivity σ_e in equation (15).

The 90° deflection time of an electron of velocity v_e by collision with ions of electric charge $q_i = e$ is given by

$$\tau_{e-i} = \frac{m_e^2 v_e^3}{8\pi n_i e^4 \ln \Lambda}, \quad (\text{B1})$$

where $\ln \Lambda$ is a quantity determined by the cut-off of the impact parameter for the collision (Spitzer 1962). For v_e we take the thermal velocity of electrons. The collision time of an electron with the neutrals is given by

$$\tau_{e-n} \approx \frac{1}{[n(\text{H}_2) \sigma(\text{e-H}_2) + n(\text{He}) \sigma(\text{e-He})] v_e}, \quad (\text{B2})$$

where $\sigma(\text{e-H}_2) \approx 6.6 \times 10^{-16} \text{ cm}^2$ and $\sigma(\text{e-He}) \approx 2.9 \times 10^{-16} \text{ cm}^2$ are the collision cross sections of an electron with an H_2 molecule and an He atom, respectively, at $T \approx 10 \text{ K}$ (Sano et al. 2000). The collision time τ_e of an electron with ions or neutrals is given by

$$\frac{1}{\tau_e} = \frac{1}{\tau_{e-i}} + \frac{1}{\tau_{e-n}}. \quad (\text{B3})$$

From equations (B1) and (B2) we have

$$\frac{\tau_{e-i}}{\tau_{e-n}} \approx \left(\frac{n_i/n_H}{1 \times 10^{-10}} \right)^{-1} \left(\frac{\ln \Lambda}{40} \right)^{-1} \quad (\text{B4})$$

for $T \approx 10 \text{ K}$. Thus, the electric conductivity σ_{\parallel} is significantly affected by collision of electrons with ions only when $n_i/n_H \gtrsim 1 \times 10^{-10}$, or at $n_H \lesssim 1 \times 10^8 \text{ cm}^{-3}$ as seen from Figure 1. Although charged grains also scatter electrons, their contribution does not affect this conclusion much because $n(\text{g}^+) \ll n(\text{g}^-) \lesssim n_i$ at $n_H \lesssim 1 \times 10^8 \text{ cm}^{-3}$, $[n(\text{g}^-) + n(\text{g}^+)]/n_H \lesssim 1 \times 10^{-10}$ at $n_H \gtrsim 1 \times 10^8 \text{ cm}^{-3}$, and $\ln \Lambda$ for electron-grain collision is not much different from that for electron-ion collision.

Similarly ions are scattered by charged grains. This effect is important compared with their collision with the neutrals only when $[n(\text{g}^-) + n(\text{g}^+)]/n_H \gtrsim 2 \times 10^{-9}$, which is not realized as seen from Figure 1. Collision of ions with electrons has negligible effect on the motion of ions unless $n_e/n_H \gtrsim 4 \times 10^{-7}$. Collision of charged grains with ions is also much less effective than their collision with the neutrals.

Although σ_{\parallel} is greatly affected by the scattering of electrons by ions at $n_H \lesssim 10^8 \text{ cm}^{-3}$, σ_P and σ_H are hardly affected as can be confirmed in the following way. Even if the effect of collision

with ions is taken into account on τ_e at $n_H \lesssim 1 \times 10^8 \text{ cm}^{-3}$, we find $|\tau_e \omega_e| \gg \tau_i \omega_i > 1$ as far as $n_i/n_H \ll 10^{-6}$, which is satisfied at least in the density range covered by this paper. Therefore, contribution of electrons to A_1 and A_2 (or σ_P and σ_H) is still much smaller than that of ions as can be confirmed from equations (6) and (7). Our results are therefore not affected by the correction on τ_e . On the other hand, we have used σ_{\parallel} with the corrected τ_e in the estimation of the Ohmic dissipation time $t_{\text{od}} \approx 10^{15} \text{ yr}$ at $n_H \approx 10^5 \text{ cm}^{-3}$ in §1; without this correction, t_{od} at this density will be 10^2 times larger.

C. On the Term “Ambipolar Diffusion”

Ambipolar diffusion is a term originally used in plasma physics. At length scales larger than the Debye shielding length in plasmas, electrical neutrality is well established. Therefore, if there is a gradient in the electron density, they are transported with a flux proportional to their density gradient, and ions follow them because of electric forces, and vice versa. Thus, particles of both electric charges diffuse as one unit. This process is called ambipolar diffusion (e.g., Cap 1976), whose literal meaning fits the process.

In the drift of magnetic fields in molecular clouds investigated first by Mestel & Spitzer (1956), ions and electrons, which are well frozen to magnetic fields, drift with the same velocity as if they were one unit. Because of this seeming similarity this process got the name “ambipolar diffusion”. However, the drift is driven by the magnetic force, not by the density gradient. Thus, this process is quite different from what the physical term “diffusion” means.

In the original ambipolar diffusion the diffusion coefficient perpendicular to magnetic field lines is inversely proportional to both B^2 and the collision time of electrons with neutrals, τ_e , when $\tau_i \omega_i \gg 1$ (Cap 1976), and thus the diffusion time is proportional to $\tau_e B^2$. On the other hand, the drift time of magnetic fields given by equation (21) in the same situation $\tau_i \omega_i \gg 1$, $t_B \propto \tau_i^{-1} B^{-2}$, has the opposite dependences. This difference also shows that these processes are quite different from each other.

Because of these differences the latter process was sometimes called “plasma drift” instead of ambipolar diffusion (e.g., Spitzer 1978; Nakano 1984; Mestel 1999).

In this paper we have found that grains, the main contributors to the frictional force, of opposite charges drift in opposite directions except at relatively low densities. This is even contrary to the literal meaning of ambipolar diffusion. Some appropriate term is desired; plasma drift is much better than ambipolar diffusion.

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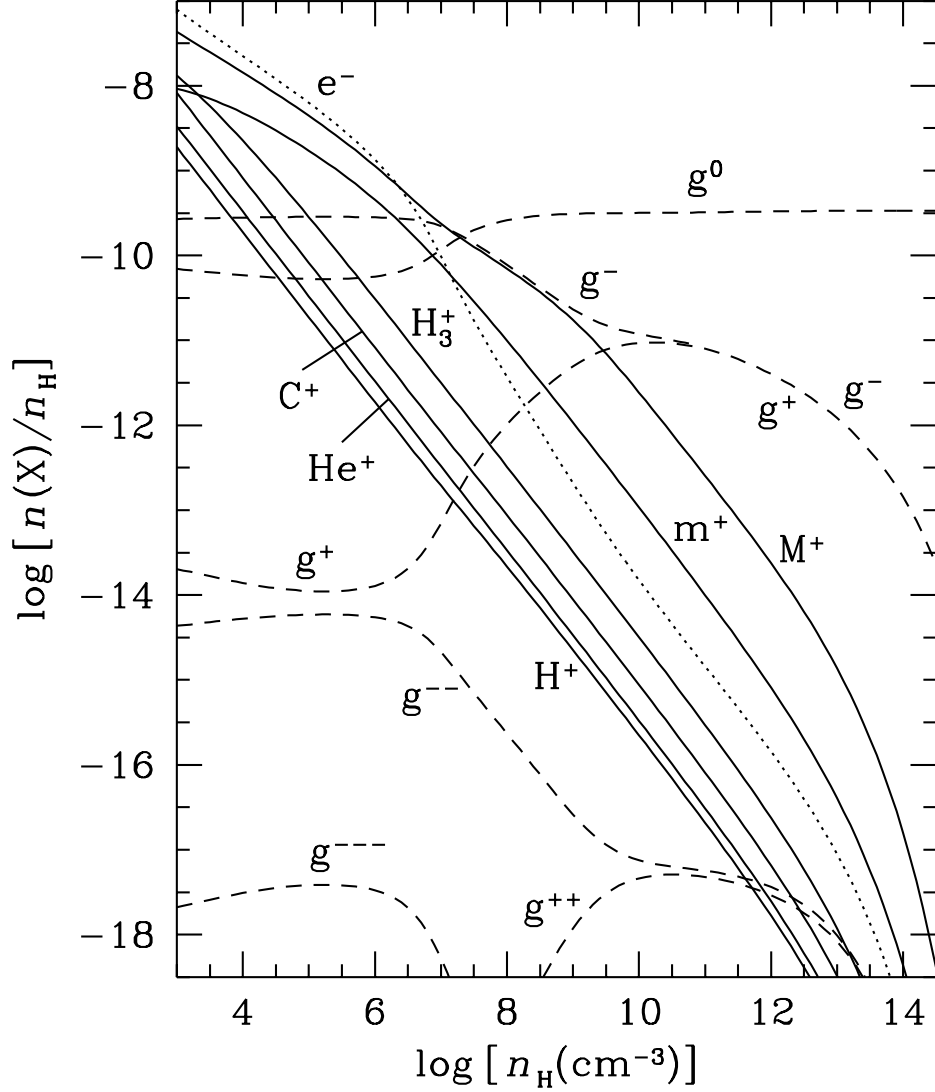


Fig. 1.— Abundances of various particles, $n(X)/n_H$, as functions of the density n_H of the cloud by number of hydrogen nuclei. The solid lines are for ions, and the dotted line is for electrons. The dashed lines labeled g^x represent number densities relative to n_H of grains of charge xe summed up over the radius. We have taken the ionization rate of an H_2 molecule by cosmic rays outside the cloud, $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$ (standard case). We have assumed that 20% of C and O and 2% of metallic elements remain in the gas phase and the rest in grains.

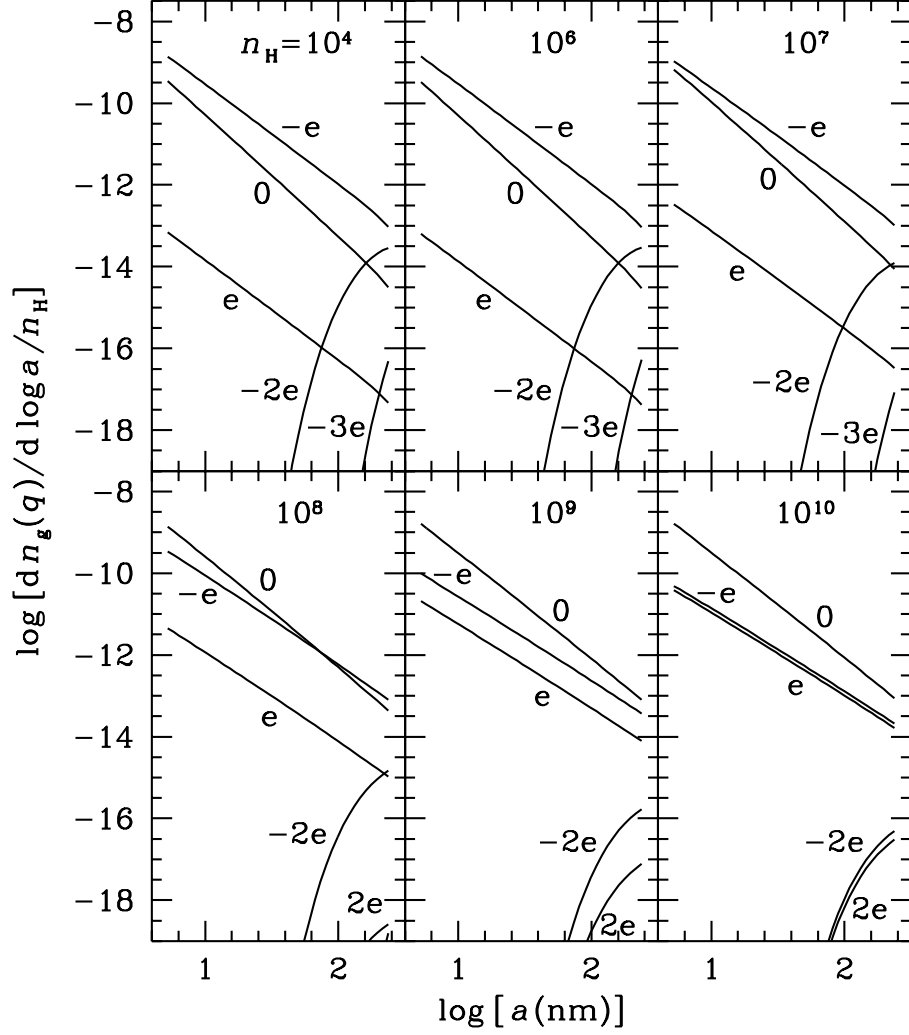


Fig. 2.— The charge state distribution of grains as a function of the grain radius a at several cloud densities for the same parameters as in Fig. 1. Each panel is labeled with n_{H} in cm^{-3} , and each line is labeled with the grain charge q_{g} .

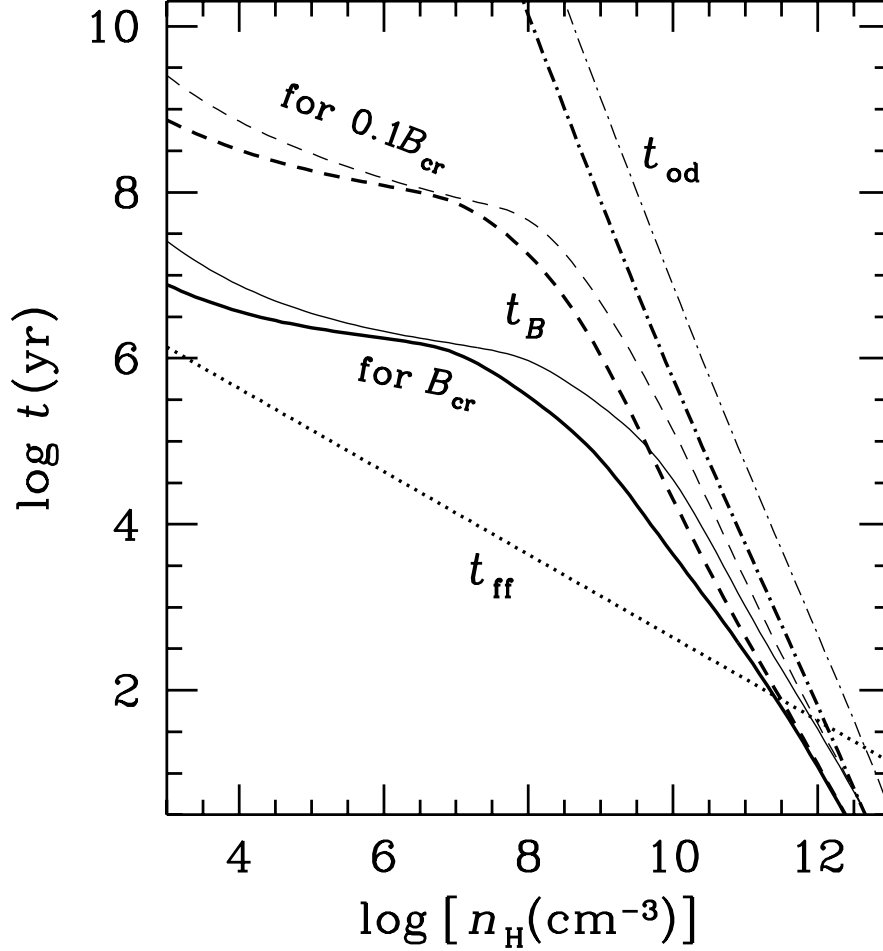


Fig. 3.— Time scales of magnetic flux loss for the clouds in which force balance approximately holds along field lines, or B_{cr} is given by equation (30). The flux loss time t_B is shown for the two cases of field strength $B = B_{\text{cr}}$ (*solid lines*) and $B = 0.1B_{\text{cr}}$ (*dashed lines*). The Ohmic dissipation time t_{od} is shown by the dot-dashed lines. Two cases of the ionization rate by cosmic rays, $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$ (*thick lines*: standard case) and $1 \times 10^{-16} \text{ s}^{-1}$ (*thin lines*), are shown. The other parameters are the same as in Fig. 1. For comparison the free-fall time $t_{\text{ff}} = (3\pi/32G\rho)^{1/2}$ is shown by the dotted line.

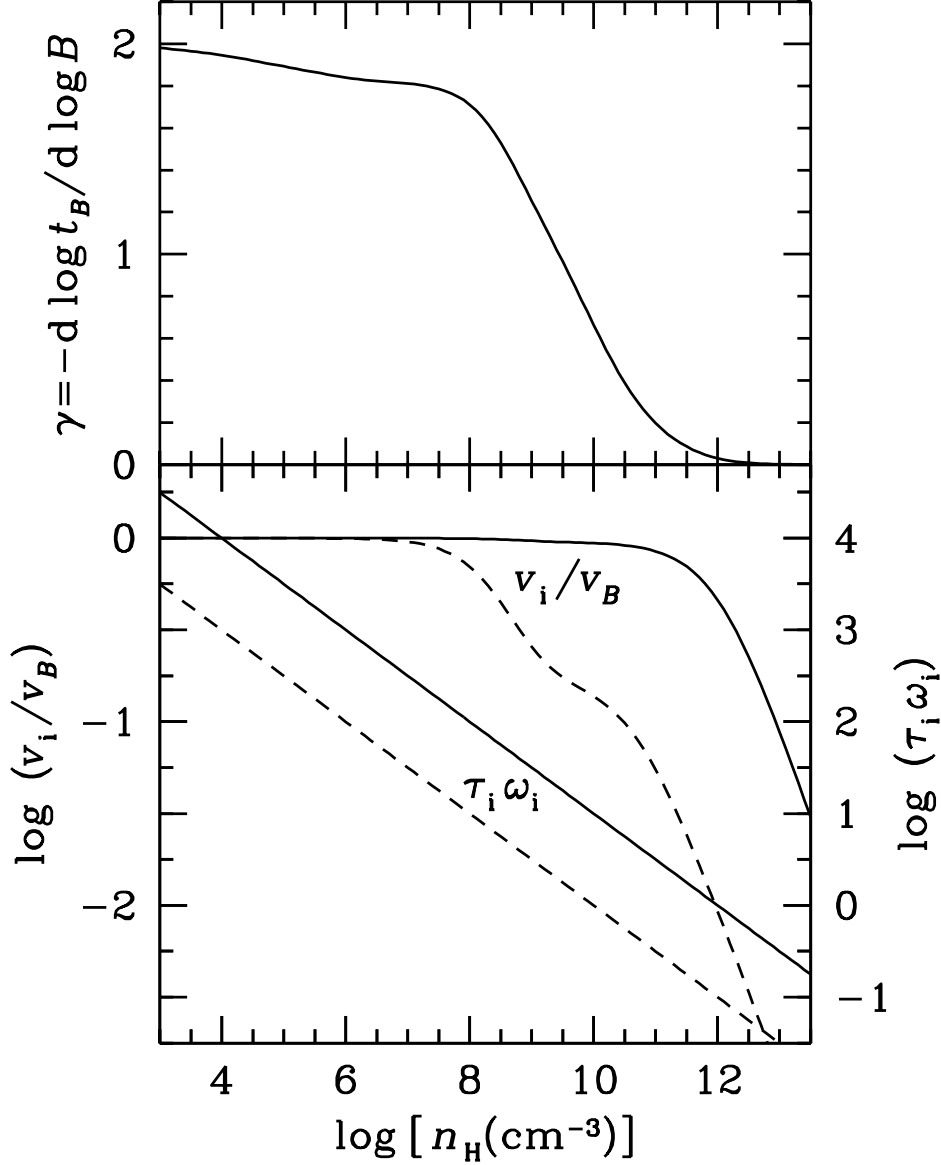


Fig. 4.— Dependence of the magnetic flux loss time t_B on the field strength B and the drift velocity of ions for the standard case $\zeta_0 = 1 \times 10^{-17} \text{ s}^{-1}$. We approximate t_B by a power law $t_B \propto B^{-\gamma}$. *Top:* The power index γ obtained by comparing t_B for the two cases $B/B_{\text{cr}} = 1$ and 0.1 shown in Fig. 3, $\gamma = -\Delta \log t_B / \Delta \log B$, as a function of the cloud density. For weaker magnetic fields we would obtain smaller values of γ . *Bottom:* The drift velocity in the direction of magnetic force relative to that of magnetic fields, v_i/v_B , given by equation (13), and $\tau_i \omega_i$ of metallic ions M^+ , dominant among various ions, for the two cases of field strength $B = B_{\text{cr}}$ (*solid line*) and $B = 0.1 B_{\text{cr}}$ (*dashed line*). We have omitted the subscript x to the velocities. These quantities take almost the same values for molecular ions m^+ other than H_3^+ , abundant next to M^+ , because their mean mass is not much different from that of M^+ .

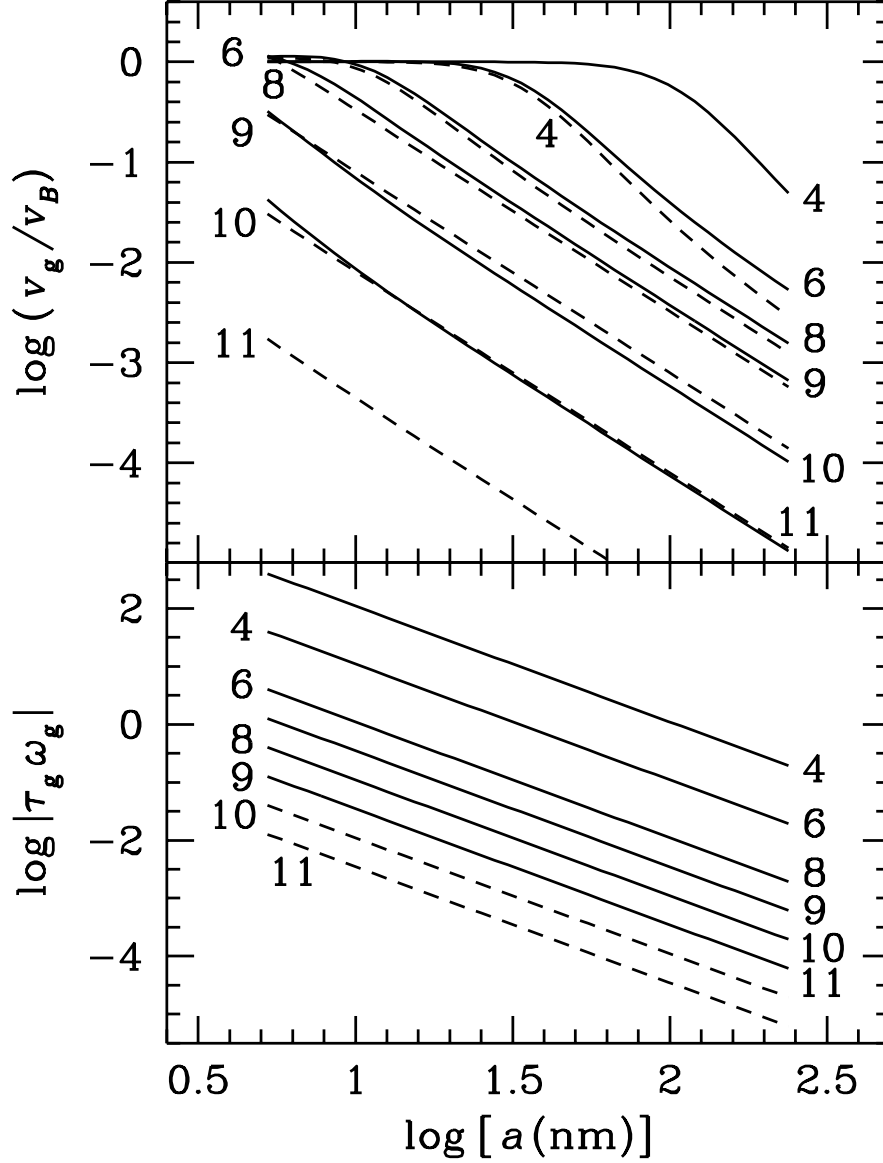


Fig. 5.— The drift velocity relative to that of magnetic fields, v_g/v_B (*top*), given by equation (13), and $|\tau_g \omega_g|$ (*bottom*) for grains of charge $-e$ as functions of the grain radius a at several densities for the same case as in Fig. 4. The two cases of field strength $B = B_{\text{cr}}$ (*solid lines*) and $B = 0.1B_{\text{cr}}$ (*dashed lines*) are shown though the dashed lines for $\tau_g \omega_g$ overlap with the solid lines except for $\log n_{\text{H}} = 10$ and 11 in cm^{-3} . The values of $\log n_{\text{H}}$ are attached to the right ends of the lines for $B = B_{\text{cr}}$ and mostly to the left ends for $B = 0.1B_{\text{cr}}$.

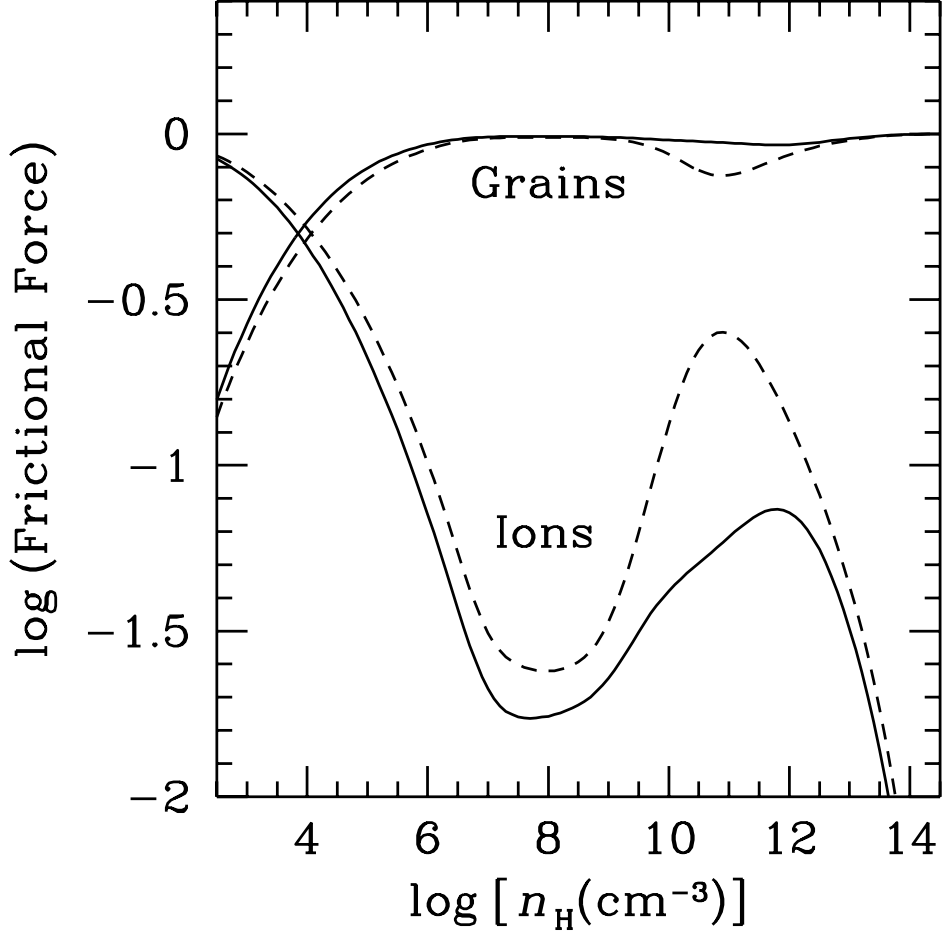


Fig. 6.— Frictional forces exerted by ions and by grains on the neutrals relative to the total frictional force, which is equal to $|\mathbf{j} \times \mathbf{B}|/c$ per unit volume, as functions of the cloud density n_{H} for the same case as in Fig. 4. Each kind of particles exerts the frictional force given by each term of equation (11). The frictional forces were summed up for all kinds of ions and for grains of all radii and all charges. Shown are the two cases of the field strength $B = B_{\text{cr}}$ (*solid lines*) and $B = 0.1B_{\text{cr}}$ (*dashed lines*). The frictional force exerted by electrons is more than 3 orders of magnitude smaller than that by ions.

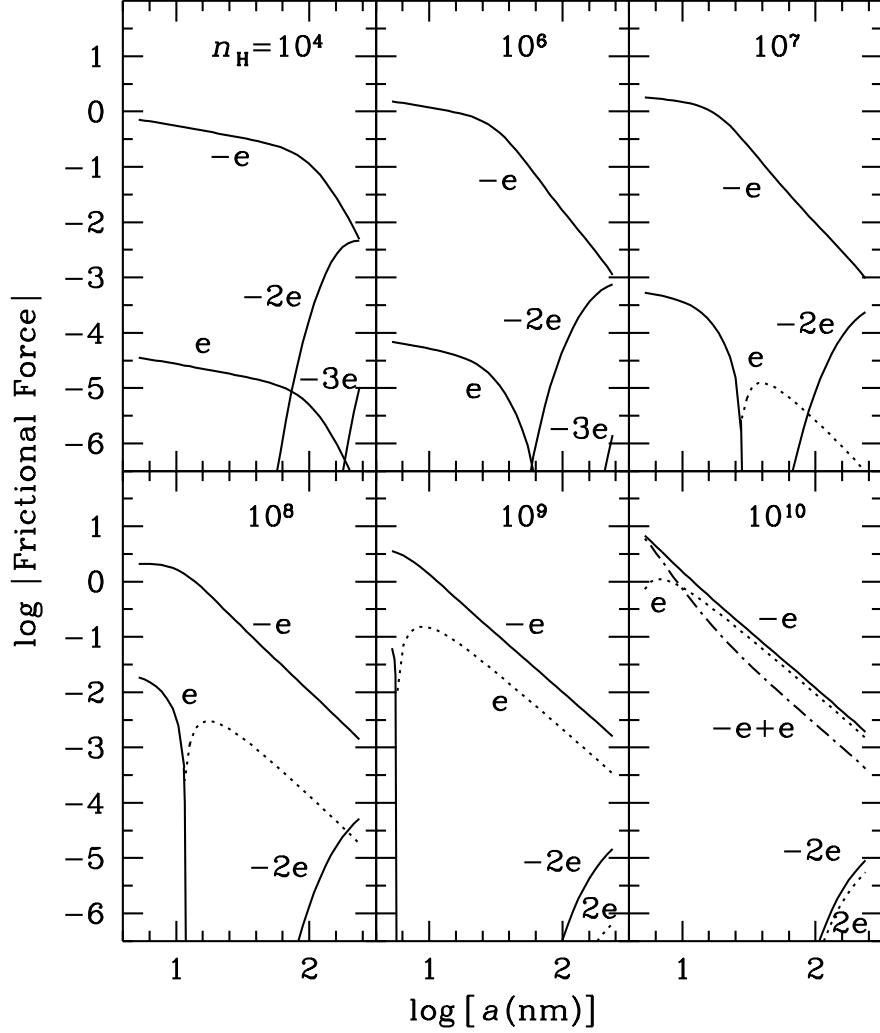


Fig. 7.— The frictional force exerted by grains on the neutrals as a function of their radius a and electric charge q_g at several cloud densities for the same case as in Fig. 4 with $B = B_{\text{cr}}$. Each curve shows the frictional force per unit logarithmic radius width $\Delta \log a = 1$ relative to the total frictional force, $|\mathbf{j} \times \mathbf{B}|/c$ per unit volume. Each panel is labeled with the density n_{H} in cm^{-3} , and each line is labeled with the grain charge q_g . The *solid lines* are when the frictional force is parallel to the magnetic force, and the *dotted lines* are when it is anti-parallel to the magnetic force, apart from the component perpendicular to $\mathbf{j} \times \mathbf{B}$. The *dot-dashed line* in the panel of $n_{\text{H}} = 10^{10} \text{ cm}^{-3}$ shows the sum of the frictional forces exerted by grains of charge $-e$ and e .

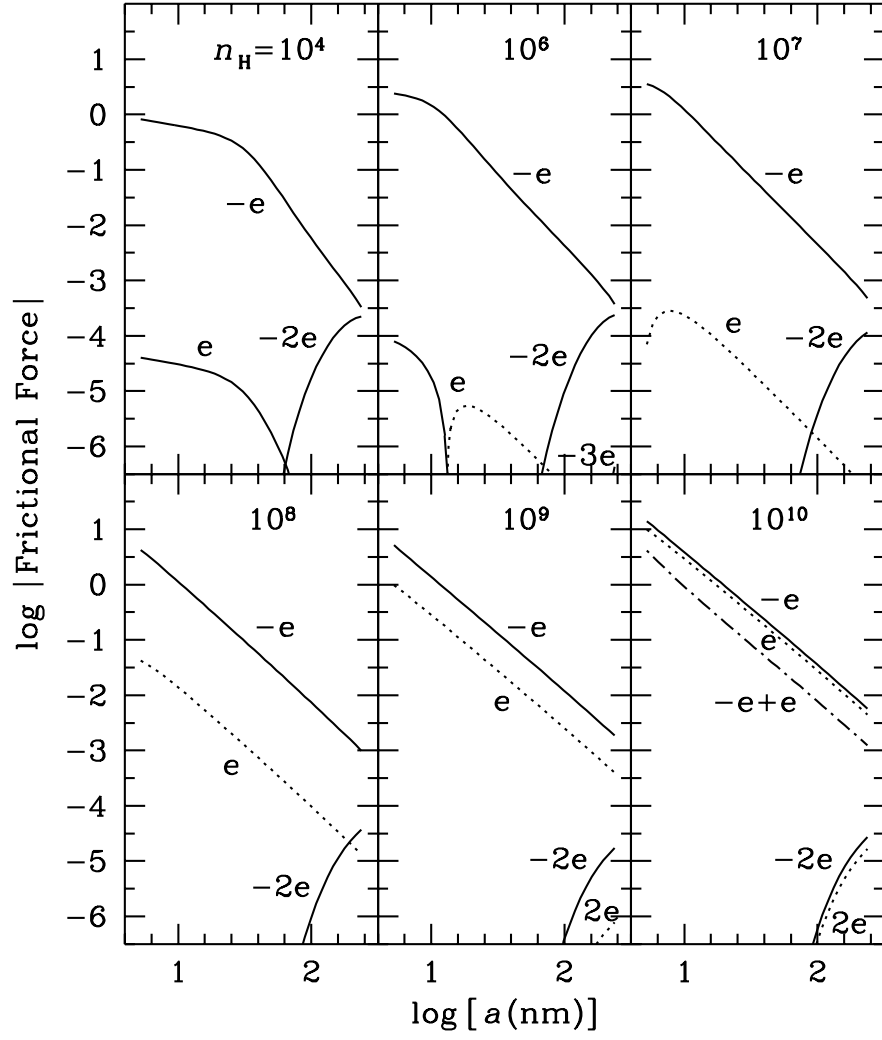


Fig. 8.— Same as Fig. 7 but for $B = 0.1B_{\text{cr}}$.

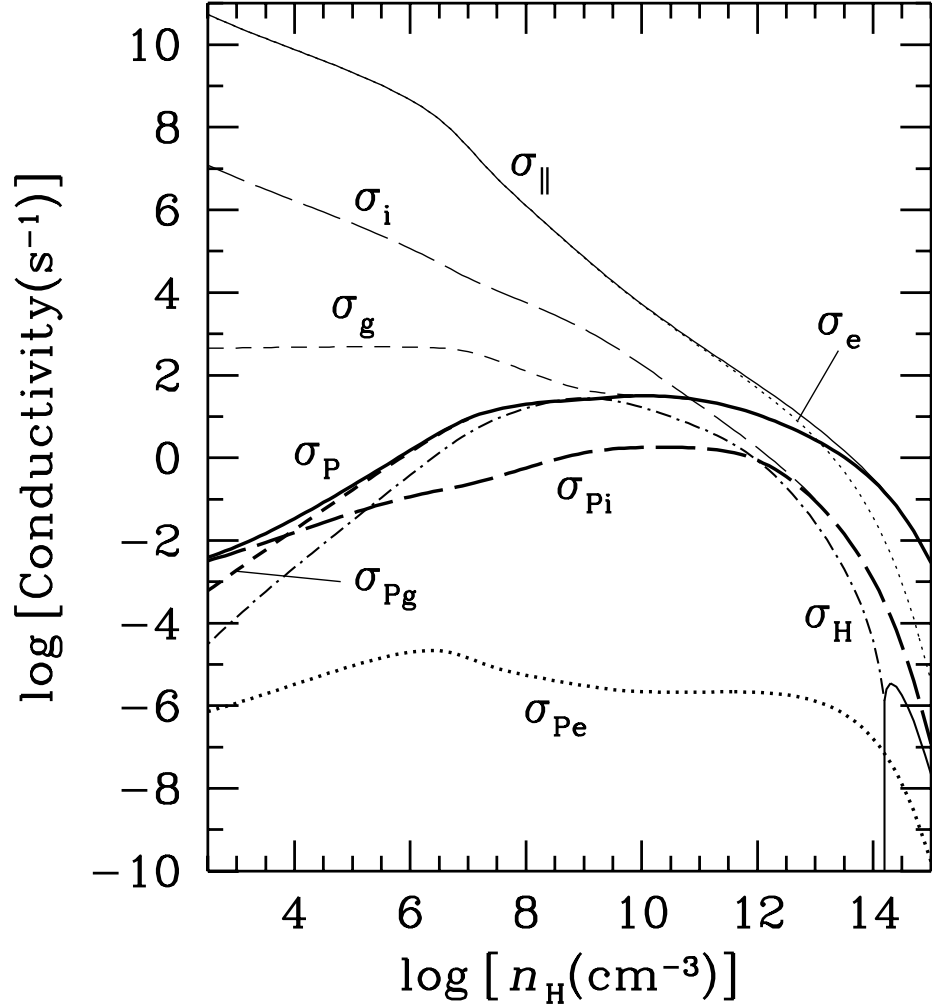


Fig. 9.— Electric conductivities as functions of the mean density of the cloud, n_H , for the same case as in Fig. 4. The thin lines are for conductivity σ_{\parallel} along magnetic field lines, and the thick lines are for Pedersen conductivity σ_P for the case of $B = B_{\text{cr}}$. Contributions of electrons, ions, and grains are shown by dotted, long-dashed, and short-dashed lines, respectively, and the totals are shown by solid lines. The line labeled σ_H represents absolute values of Hall conductivity for the case of $B = B_{\text{cr}}$; the dot-dashed line is where $\sigma_H < 0$, and the solid line with $\sigma_H > 0$.